

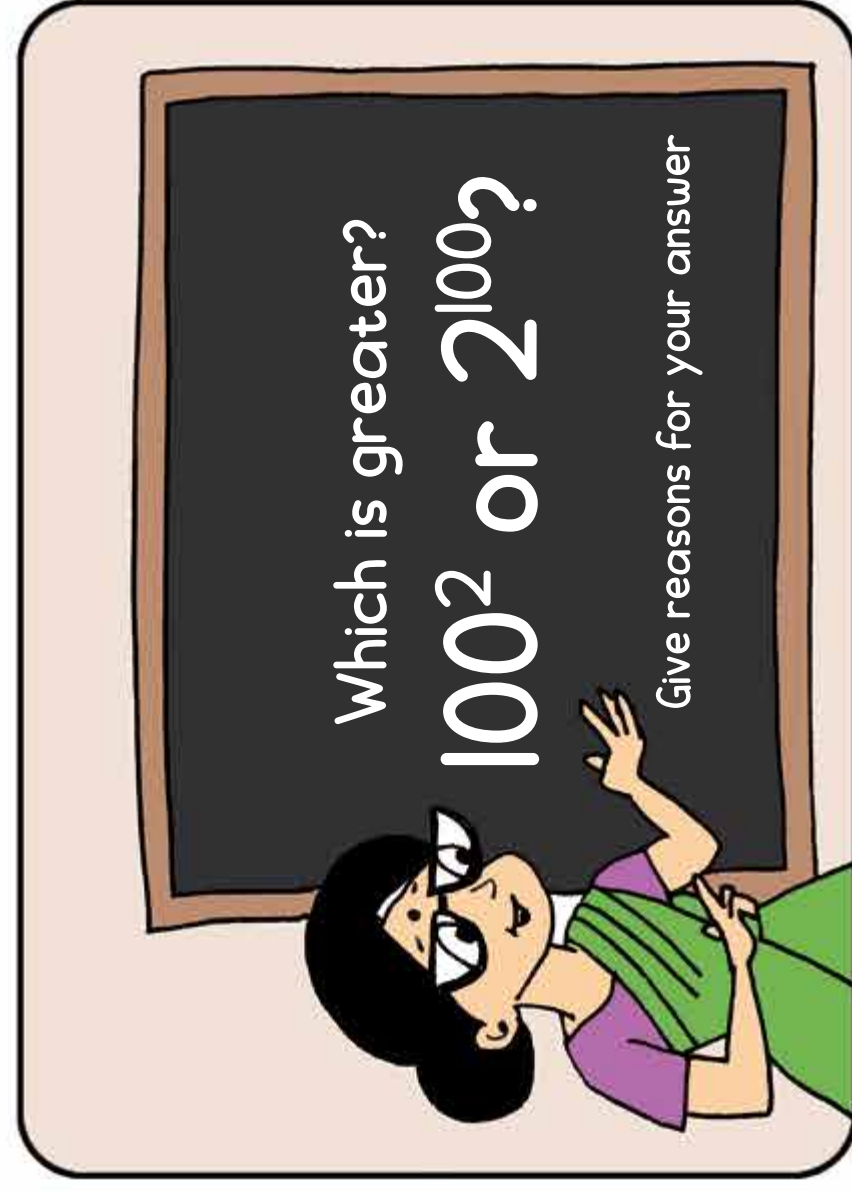
LITTLE MATHEMATICIANS

In the previous article, you read about children's algorithms for problem solving. Here, teacher Krittika Hazra shares her experience of the same. You are sure to be blown away by how these children reasoned. Note her probing questions which stimulate the child to reason, defend or backtrack on the argument presented.

Listen to Krittika: Has this ever happened to you? You have posed a question to the class and are moving around checking their answers. And then suddenly something happens. You pause, turn back to a page already checked and look at the answer once again. You wonder - how has this child arrived at this answer? This does not quite match the method you had used! And yet, it is flawless!

This is exactly what happened to me a few days back. I was amazed by the originality of the methods taken by the children and their responses to my questioning. Take a peek into my classroom.

This was my question ... >



A 100^2 can be calculated. 2^{100} cannot be calculated. If 2^{100} cannot be calculated, that means it is too big to calculate. Hence $2^{100} > 100^2$.

I am simply in love with this spontaneous answer.

Is everything that is too big to calculate impossible to calculate? Do you know of anything that has been calculated despite being too big?

Yes, there are many things that have been calculated by scientists despite being too big. Distance between Earth and planet Mars is 54.6 million kilometers. The mass of Earth is 5.9736×10^{24} kg. The diameter of earth is 12,756 km.

If everything that is too big was dropped from calculation, where would we be? What information would we miss?

If everything that was too big was dropped from calculation then we would be in the dark about the existence of the universe, about the distance between the planets. We would not be aware of the old civilizations like the Harappa and Mohenjodaro. The fossils that are available now help us know about the old flora and fauna.

B The values of 2^{100} and 100^2 are same because the values of 2^4 and 4^2 are the same. Because if we exchange the exponents and bases, the values remain the same!

Can you use the same argument for 3^4 and 4^3 ?

C $100^2 = 10^4 = (10^2)^2 = 10^4$
 $2^{100} = (2^{10})^{10} = (10^2)^{10} = 10^{20}$
 Hence, $2^{100} > 10^4$
 So, $2^{100} > 100^2$ (Ans)

D I used the idea of $(a^n)^m = a^{nm}$ and broke the bases as required.

How simple and elegant!

We know that 2^{63} is greater than 100^2 !
 How did you know this!!!!!! Why did you use 2^{63} ?

Actually before this test you had shown a video on which a farmer requested grains for each square of a chessboard and on the last square, the number of grains came to 2^{63} . So from there I had multiplied in a rough sheet of paper.

How did you know 2^{63} is greater than 100^2 ?

As we know that $2^7 = 128$. So 2^{14} will be 128×128 which is obviously more than 100^2 . But here I knew that what 2^{63} is so I used it.
 Yes we can, but it will take us longer. We can reduce 2^{100} to 2^{14} and then calculate to conclude.

E Since 100 is greater than 100, and in the question 2 has a greater power than 100, hence $2^{100} > 100^2$.

You meant as the exponent 100 is bigger than exponent 2, that is why 2^{100} is bigger. Will it always work?

Yes, I have checked many examples and they are working. For example - 8^8 and $9^8 \rightarrow 8^8$ is greater; 11^{13} and $13^{11} \rightarrow 11^{13}$ is greater; 125^{100} and $100^{125} \rightarrow 100^{125}$ is greater.

F 0^{100} (or any no.) and 100 (or any other no.)⁰ - In this case the rule does not apply because $m^0 = 1$, $0^m = 0$, $m \neq 0$. And therefore $100^0 > 0^{100}$. I ALSO FOUND A PAIR $\rightarrow 2^4 = 4^2 = 16$ (though I did not find any other pair like this till now.)

How did you know that 2^{100} is a proper fraction i.e. less than 1?

Since $\frac{100^2}{2^{100}} < 1$ Hence $100^2 < 2^{100}$.

A proper fraction has the smaller number on top and the greater number in the denominator.
 I know, I also mentioned that the base has to be too small and the exponent has to be too big.

How do you know 2^{100} is the greater number? Because in 100^2 the exponent is smaller than the base. But in 2^{100} the base is too much smaller than the exponent? How about 2^3 and 3^2 ?

Okay so you mean if the base is small and the exponent is big, then your logic works. Can you find out a general rule? What should be the minimum difference between the base and exponent to satisfy your rule?

G $2^{100} = (2^{10})^{10} = (1024)^{10}$. Hence $1024^{10} > 10000$ which is 100^2 .
 Why did you calculate $(2^{10})^{10}$? How did you know $(1024)^{10}$ is bigger than 100^2 ?

First I found $(2^{10})^{10}$ to make it easy to calculate 2^{100} . The base number 100^2 and the number which came was 10,000. Since 1024 had the power 10 and 10,000 had no power so I could conclude that $(1024)^{10}$ will be a lot greater than 10,000.

H Is it because 2 is getting multiplied too many times!!??

$2^{100} = 2 \times 2 \times 2 \times 2 \dots$ so on. Hence 2^{100} is obviously greater than 100^2 .

$100^2 = 10,000$
 $2^{100} = 2 \times 2 \times 2 \times 2 \dots$
 $= (2 \times 2) \times (2 \times 2) \times (2 \times 2) \dots$
 $= 4 \times 4 \times 4 \dots$ Therefore we can conclude $2^{100} = 4^{50}$.
 In the same way we can also say that $4^{50} = 16^{25}$.
 Now we have to say, which is greater: 100^2 or 16^{25} .
 Let us just take $16^4 = 16 \times 16 \times 16 \times 16 = 65,536$.
 And $100^2 = 10,000$.
 Therefore, $16^4 > 100^2$.
 Therefore, as the smaller unit ($65,536$ or 16^4) is greater than $10,000$ or 100^2 , we can conclude that the greater unit (16^{25}) is also greater than $10,000$.

Why did you calculate $(2^{10})^{10}$? How did you know $(1024)^{10}$ is bigger than 100^2 ?