Games and Explorations in Mathematics

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Who taught us how to use a smartphone? Didn't we learn by playing with it? Some of us are still learning!

Many have argued the importance of play when it comes to children's learning. We are all familiar with quotes like 'play is the work of childhood'. When it comes to children, in particular, we are beginning to blur the dichotomy of 'work' and 'play'. But why is play so important? How does it help children (and adults) learn? What role does it have in the teaching and learning of maths in particular? Let us start with what we mean by 'play' in this article.

Structured play or games

One way to look at 'play' would be to think of it as 'games' with a clear win or loss (or draw); it can be played with one or more players, in teams, or just by oneself (like *Solitaire*). These necessarily involve rules, can be dictated by chance or probability (especially if a game involves dice and/or cards) and/or is strategy-driven (like chess) or a combination of luck and strategy (like bridge). *Jodo Gyan* and other resource groups use several modified dice for fractions, shapes etc. There are some card games, as well.

Since the end goal of this kind of play is clearly a win, proficiency is desirable and high in demand. So, games naturally provide the player with a purpose to sharpen his/her skills. This can be harnessed by any subject, especially maths, where computational proficiency is important. The usual prescription for mastering computations is drill and practice, which is undoubtedly boring and does cause children to lose interest in the subject. However, if the same is converted into a game – to be played as a team, then the same goals can be achieved with much more motivated participation from the learners. Moreover, peer learning can be catalysed since teams will put in all the effort to improve their performances. But the teacher or facilitator needs to set things up so that each child gets better at it.

The simplest way is to create two (or more teams) and quiz them periodically. The actual quiz can be timed, as in, questions have to be answered immediately or within a few minutes. It can be designed so that the first team to get it hits a buzzer. This should include some penalty for the wrong answer since children should not rush and compromise accuracy for speed. Or it can be designed in such a way that each team reveals their answer at the end of the allotted time. The need for penalty is less in this case, and therefore, the win is more because of one's own calibre rather than the mistake(s) of the other team(s). Therefore, this encourages proficiency but does not make the game too competitive. Also, the topic of such a quiz can be declared in advance and the teams given enough time to practice on their own, which should be supervised by the teacher (acting as the coach).

Team selection is critical. It is recommended that each team consist of a mix of children – those who are proficient and those who are yet to get there. It is also critical to ensure participation by each member of the team so that the team invests in improving the performance of each member and not rely only on its star players.

As children are learning addition-subtraction (and later multiplication-division) it is very helpful if they automatise certain number facts, for example, which numbers add to ten, 1 + 9, 2 + 8, 3 + 7, 4 + 6 and the sum of any two single-digit numbers, for example, 7 + 8, 6 + 5 etc. (and later 8 × 6, 7 × 9 etc.). Rapid-fire rounds can motivate children to automatise these number facts. However, to ensure that this is not just rote memorisation, and to check that the children know how to arrive at these results, the quiz can include bonus points where the child (or the team) giving the right answer is asked to rationalise their answer. For sums involving computation with numbers < 100, the bonus can be enriched further: In how many ways can you solve 56 + 37? where each method fetches a point. This fosters number sense as they will be playing with the numbers beyond (and ideally before getting into) standard algorithms. It helps them come up with their own shortcuts for mental maths.

Note that this playing with numbers is quite different from its 'game' avatar. We will return to this after discussing a few more aspects of games.

The rule-based world of a game directs the players to operate and think within certain set limits. So, one has to be creative within the given bounds. This facilitates navigating the world of maths, which is rule-based but allows many adventures within the same. It is especially a good idea to let children explore what is possible and what is not within the rules of a game, (for example, in chess, which squares can the black knight occupy) and

within maths (for example, is it possible to find two odd numbers that add up to another odd number? Or if a quad is a kite and a trapezium, what are its properties?). Strategy-based games demand higher-order thinking skills, a deeper level of understanding and possibly some creativity. The Random Digits Game (check references) is one such.

Games can also be utilised for assessments. Instead of asking for definitions, the teacher can ask for examples and ideally, non-examples. But the 'game' aspect can be utilised even further. Once children get the hang of a certain type of tasks, they can be asked to create similar problems/tasks specifically to challenge the other team(s). This way, children would be motivated to create complex questions allowing their creativity a chance to flourish.

Free exploration play

Now let us come to the other avatar. Play also means free exploration which is not bound by any rules, or the rules can be developed if and when needed. Young children learn a lot through role-play, alone or with others (children and/or adults) where they imitate (mostly) adults and learn by experiencing the lives of people around them (or of the characters they know). Another aspect of this 'exploration' kind of play also involves familiarising oneself with a certain situation or material and developing an understanding of the possibilities as well as the boundaries. When a child explores origami, for example, she or he learns how paper behaves, what can be done, what is difficult to do (and what is impossible). Unlike games, the end goal is usually not clearly defined in exploration, at least not at the beginning, and can evolve as the explorers' understanding of the situation or the material grows. Both kinds of play, games and explorations, are important when it comes to learning maths and both should be utilised.

Starting right

Before we move ahead, let us take a closer look at this school subject which is not only a discipline but also a form of understanding. The image of maths in the minds of most people – parents, teachers and therefore, children – is that it is a rule-driven, dry subject strictly hierarchical with very little or zero room for creativity; you either get it or you do not. On the contrary, maths can and should be discovered by children – starting from class I (or earlier). Once the basics are taught, that is, the meaning for various number names by relating them to their respective quantities, the numerical representations of numbers, place-value or more importantly, the idea of bundling in tens and how that is crystalised as numerals, children should be able to figure out more-less, and come up with the rules to find the smaller/smallest (or bigger/biggest) number given a bunch of numerals (for example, order these numbers from smallest to largest: 38, 83, 40, 9).

It is crucial that children have access to some manipulatives (bundle-sticks and/or *Ganitmala*) to make these numbers and then decide which indicates the smallest quantity (no bundle) and which indicates the largest (highest number of bundles). These explorations or playing with numbers can be done in groups and teachers can push the children to articulate their findings and consolidate that understanding into a rule. This exercise will help develop mathematical communication and help students appreciate the precise as well as concise nature of the mathematical language. *Meaning of Fractions* (video link in references) illustrates how a teacher can get children to articulate by asking questions and not giving away the answer.

Explorations and investigations

Similarly, once the meaning of addition, subtraction (and later multiplication and division) has been explained and understood, different ways of adding and subtracting numbers should emerge out of children's play with quantities. Different kinds of manipulatives will help them discover different ways of computing. The algorithms can and should emerge out of such 'play'. The teacher's role is very important in helping children jot down their explorations in writing. When children get the sense that they discovered the standard algorithm themselves, it becomes highly empowering, and maths becomes something that can be learnt on one's own. In fact, this is exactly why those who love this subject, get drawn to it. More importantly, when children get the taste of this self-dependence, they no longer want to be told how to proceed, rather they want to embark on such adventures themselves. Consider the difference in teaching such children (vs the usual group) – it will be easy since the learners are eager and ready, but it will also be challenging because their hunger is of a different kind. The teacher must go beyond the usual and look for problems, explorations, investigations that suit such adventurous learners – not too easy (and therefore, boring) but challenging enough to attract and engage them. Fortunately, the internet is full of such explorations and investigations, and most (if not all) teachers can access these, thanks to smartphones. (Check references for some such resources, especially the *Thinking Skills* pull-out.)

Similarly, multiplication tables should be constructed and then various patterns in them can be explored. Many children discover the digit sum property of multiples of 9 on their own. More such discoveries are possible and can make the maths class quite vibrant and exciting. Exploring multiples of numbers on the 10x10 grid of numbers 1-100 can lead to discovering the divisibility rules.

Discovering patterns

These games and explorations need not remain within numbers and operations. Playing with base-10 blocks (Flats [hundreds], Longs [tens], Units or FLU) helps transition to the algebra tiles just as *Ganitmala* builds the mental image of a number line. The explorations can lead to a lot of insights that should be harnessed by asking children to first make conjectures and then justify them. The habit of exploration enables children to look for patterns. This can be enhanced further by encouraging them to observe maths in their surroundings and everyday lives.

To motivate them further, the 'game' avatar can be evoked – 'find squares in your neighbourhood' (rectangles and circles would be too easy) with the bonus 'why is this a square and not a rectangle'. Another investigation can be to find mathematics in various vocations/crafts – how does a carpenter or a tailor use mathematics? As the sense of geometry (shapes, as well as, symmetry and spatial understanding) and algebra grows, formulas for perimeters and areas of polygons (and circles etc.) can be derived by making the shapes with matchsticks and card cut-outs of the same. Similarly, explorations with nets of solids (cuboids, cylinders, cones, spheres etc.) lead to the understanding of surface areas in particular and help with volumes as well. Exploring tessellation with different kinds of tiles (all possible types of triangles and quads) can lead to many theorems.

Explorations can lead to many interesting observations, discoveries, and interlinkages within maths (for example, which numbers cannot be written as a sum of consecutive natural numbers? Why is the hypotenuse of the biggest triangle in a 7-piece tangram slightly smaller than thrice the side of the square?).

The National Curricular Framework (NCF) 2005 and the Position Paper, Teaching of Mathematics, emphasise the mathematisation of children's thinking. Play – like games and especially as exploration – can achieve that by integrating several of the processes with the content. No wonder textbooks are now including games. And how much can children resist playing?

References

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- Sum of Consecutive Natural Numbers: http://azimpremjiuniversity.edu.in/SitePages/resources-ara-august-2017-lfhc-sums-of-consecutive-natural numbers.aspx
- Tangrams: http://teachersofindia.org/en/ebook/getting-shape-tangram-time
- Other pull-outs, activities and related articles: https://azimpremjiuniversity.edu.in/SitePages/resources-at-right-angles.aspx
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