

# *Dynamic Geometry Software:* A Conjecture Making Tool

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Digital technologies have afforded many possibilities for the teaching and learning of mathematics and this has become a major area of research in Mathematics Education. A particular class of digital tools, known as Dynamic Geometry Software (DGS), allows mathematical concepts to be explored in a visual – dynamic way. In a DGS, geometrical figures or diagrams can be dragged and manipulated thus making them dynamic. The ‘dynamism’ of these figures provides opportunities to students to experience mathematical properties very differently from static figures which they generally experience in a traditional geometry class. In this article, we shall illustrate how middle school students experienced the angle sum property of a triangle, a fundamental geometrical idea, using GeoGebra, an open source DGS. A dynamic geometry software is also referred to as a dynamic geometry environment (DGE).



*Keywords: DGS, geometry, algebra*

## Getting started with GeoGebra

GeoGebra enables the user to explore mathematical concepts using multiple representations. The reader may wish to download the software and explore. The steps for downloading the software are given at the end of the article. As soon as a new document is opened in GeoGebra, it offers a Graphics view, an Algebra view and an Input Bar at the bottom of the window (Figure 1).

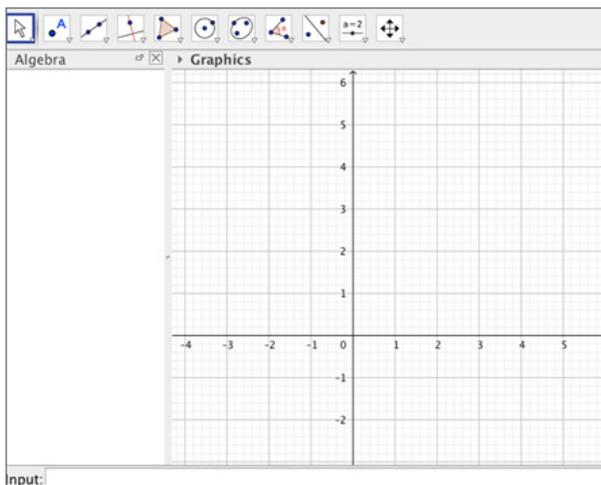


Figure 1. The Algebra view, Graphics view, and Input bar in a new GeoGebra document.

Other views can be accessed using the View option on the toolbar (see Figure 2). For example, by selecting Spreadsheet from the View option one can bring in a spreadsheet as well (see Figure 3). The Graphics view, Algebra view and Spreadsheet view can be used simultaneously to explore properties of a figure geometrically, symbolically as well as numerically.

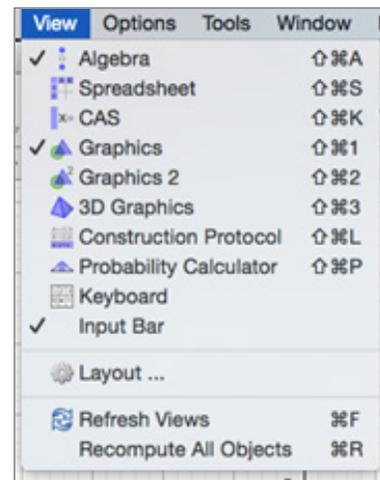


Figure 2. The View option allows the user to select different views.

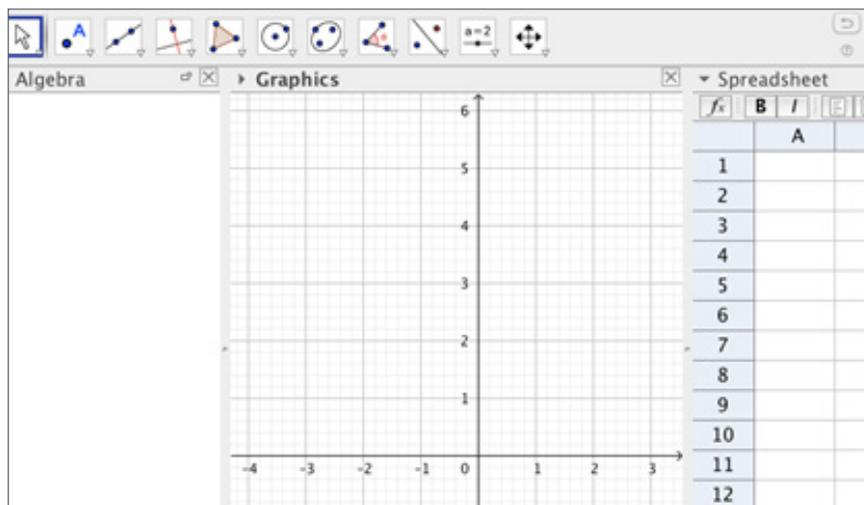


Figure 3. The Algebra view, Graphics view, and Spreadsheet view in a new GeoGebra document.

To understand this better let us draw a triangle in the Graphics view using the **Polygon tool** of the GeoGebra toolbar as shown in Figure 4. (To do this, select the Polygon icon from the tool bar and click on the Graphics view. GeoGebra will mark the first point as A. Proceed to draw a triangle by clicking on two other locations, which will be labelled as points B and C. To complete the triangle, return to point A and click on it again. This completes the triangle ABC.) GeoGebra immediately displays the coordinates of the vertices A, B, C, the lengths of the sides  $a, b, c$ , and the area of the triangle

(displayed as t1) in the Algebra view. This reiterates the point that a triangle is a two-dimensional plane figure, a polygonal region in the plane, an aspect often neglected while drawing shapes using chalk on a blackboard!

Selecting one of the vertices, say A, and dragging it on the Graphics view will lead to varying experiences of the triangle while its measurements will change dynamically in the Algebra view. Figure 5 shows an obtuse angled triangle as well as (an almost) right-angled triangle obtained by dragging.

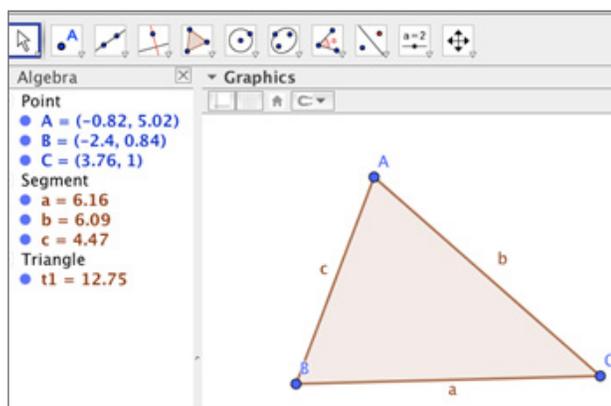


Figure 4. A triangle has been drawn using the Polygon tool whose parts are named and described in the Algebra view.

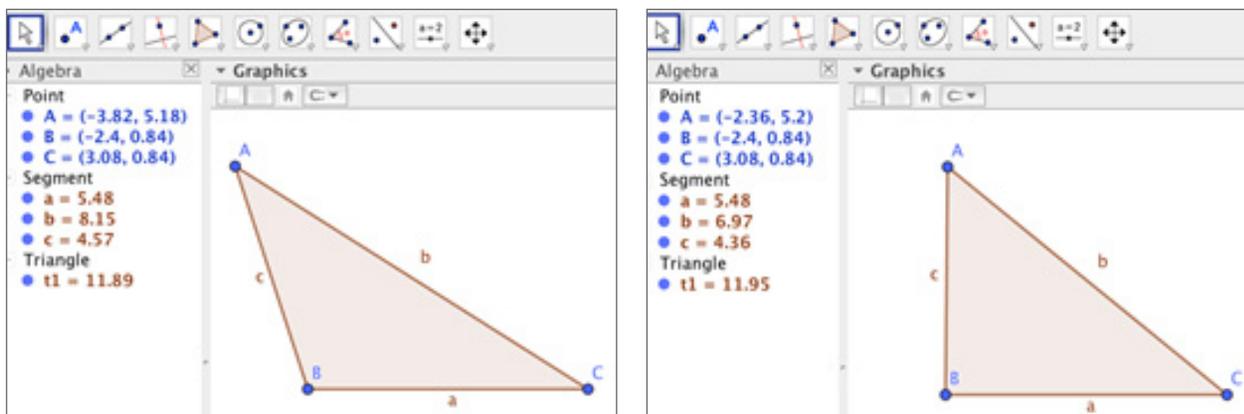


Figure 5. Varying appearances of a triangle obtained by dragging vertex A.

## Dragging as a tool

The dragging feature is perhaps the most powerful aspect of a DGS. While dragging parts of a figure in GeoGebra, the Algebra view as well as the Graphics view reveal that certain attributes of the figure vary whereas others remain invariant. According to Leung (2012)

*A key feature of DGE is its ability to visually represent geometrical invariants amidst simultaneous variations induced by dragging activities..... the variations of the moving image are perceived in contrast to what simultaneously remains invariant (p 2).*

Being able to discern what varies and what remains invariant is key to experiencing a mathematical concept or property. This important feature can be exploited to enable students to make conjectures while performing geometrical explorations.

Leung (2003) explains

*.....when engaging in mathematical activities or reasoning, one often tries to comprehend abstract concepts by some kind of “mental animation”, i.e. mentally visualizing variations of conceptual objects in hope of “seeing” patterns of variation or invariant properties (p 1).*

To illustrate this aspect let us see how grade VI students abstracted the angle sum property of a triangle using the dragging feature in GeoGebra. Thirty five grade VI students in a school in New Delhi were being introduced to GeoGebra as a tool to explore geometrical concepts in their curriculum. The fact that the sum of the three interior angles of a triangle is equal to  $180^\circ$  was demonstrated by their teacher when they studied the topic of triangles. This was done through an activity where, given different triangles, students were required to measure the three angles (using a protractor) and obtain the sum. This task led to some confusion, as the angles did not turn out to be whole numbers (for some students) and the sum was close to 180 degrees but not exactly 180 degrees. In another

activity students were given triangular cut-outs and were made to cut out the corners and bring them together to form a ‘straight line angle.’ Such visual representations were often treated as proofs in the classroom. However, it should be emphasized that such a model only helps to visualize the angle sum property and is not a proof in the mathematical sense.

The students were familiarized with the basic drawing tools of GeoGebra before they proceeded to explore the angle sum property. They worked in pairs and used the **Polygon Tool** to draw a triangle as shown in figure 5. Each pair of students dragged the vertices of their respective triangles and observed the changes in the Algebra view. In the following step, they were required to measure the angles of the triangle by using the **Angle tool**. Prior to this, they were instructed to go to **Options -> Rounding -> 0 decimal places** so as to avoid angle measures appearing in decimals. A click inside the triangle after selecting the **Angle tool** from the toolbar marked the angles inside the triangle as  $\alpha$ ,  $\beta$  and  $\gamma$  along with their respective values. The teacher instructed the students to define a variable called **anglesum** and enter the following in the **Input Bar**:

$$\text{anglesum} = \alpha + \beta + \gamma.$$

As a result of this step,  $\text{anglesum} = 180^\circ$  appeared in the Algebra view (as shown in Figure 6).

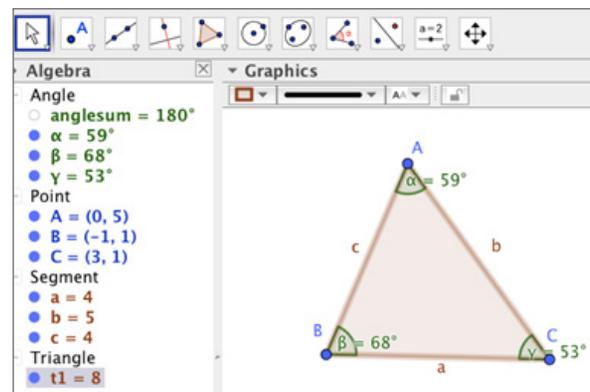


Figure 6. GeoGebra measures the sum of interior angles as  $180^\circ$  in the Algebra view.

Students were then asked to drag any vertex of their triangle and observe the Algebra view again. This time students noted that the anglesum remained constant (at  $180^\circ$ ) for each type of triangle on their screen, while other aspects varied. Their observation of the invariance of the sum of the three angles amidst variation of the side lengths, the angle measures and the area of the triangle, led them to conjecture that  $\alpha + \beta + \gamma$  is  $180^\circ$ . Although students had explored the angle sum property through other activities, GeoGebra offered an exciting and different way to explore the problem, along with the added advantage of being able to examine many triangles very quickly to develop the idea.

To add another dimension to the exploration, the teacher asked the students to bring in a spreadsheet. The values of the angle measures  $\alpha$ ,  $\beta$  and  $\gamma$  were entered in the columns A, B and C

respectively after putting the headers  $\alpha$ ,  $\beta$  and  $\gamma$  in the cells A1, B1 and C1 respectively.

Explorations using technology can often lead to surprises and a teacher using technology in her classroom must be prepared to deal with them. One of the student pairs had obtained a figure as shown in Figure 8 on their GeoGebra screen. When they clicked inside their triangle (after selecting the **Angle tool**), GeoGebra marked the reflex angles instead of the interior angles! This happens when the triangle is drawn in the anticlockwise sense. This unexpected output fascinated everyone as they eagerly tried to find out why GeoGebra produced this output. Further, they dragged the vertices of the triangle and conjectured that the angle sum (of reflex angles) remains constant at  $900^\circ$ . This was followed by a spreadsheet exploration, which confirmed their conjecture numerically. Figure 8 shows the output.

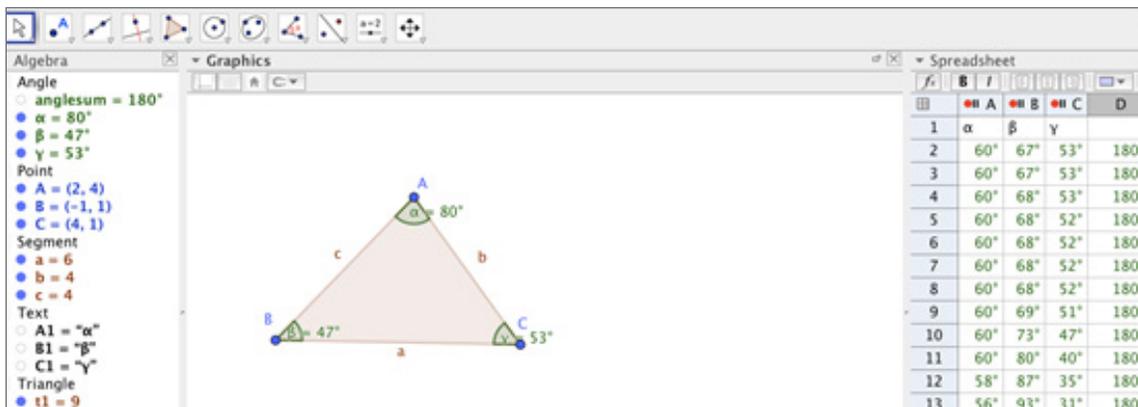


Figure 7. Algebraic, graphic and spreadsheet views illustrating the angle sum property.

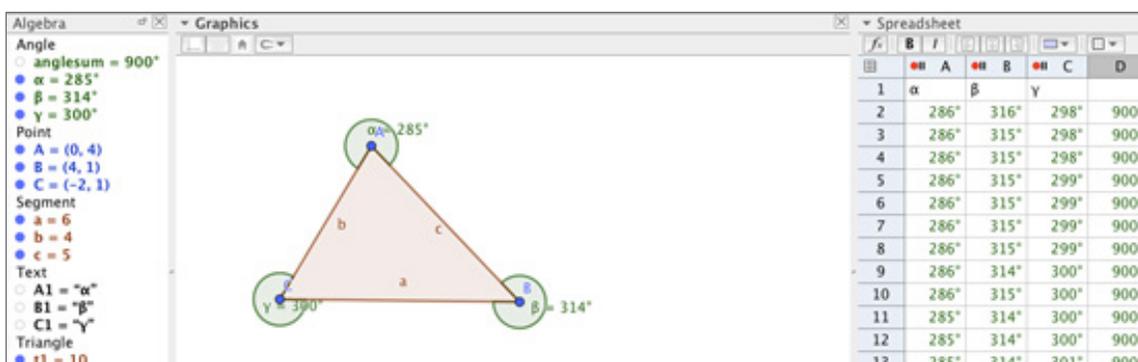


Figure 8. Algebraic, graphic and spreadsheet views, illustrating that the sum of reflex angles of a triangle is  $900^\circ$ .

Thus dragging the vertices of a triangle led to two invariants, namely the interior angle sum which is equal to  $180^\circ$  and the reflex angle sum which is  $900^\circ$ . The simultaneous occurrence of these invariants along with variations in various attributes of the triangle, such as the changing side lengths, coordinates of vertices and area was the key in making conjectures regarding the angle sum properties. The teacher asked the class to justify the new finding (regarding the reflex angle sum). Is the sum of reflex angles always  $900^\circ$ ? If so, why? Can you explain? In order to scaffold students' thinking, she pointed out that at any vertex of the triangle, say at A, the sum of the interior and reflex angles is equal to  $360^\circ$ . Students carried forward the reasoning and arrived at the following explanation.

The total sum of interior and reflex angles of a triangle is  $3 \times 360^\circ = 1080^\circ$ . The interior angle sum is  $180^\circ$ , thus the reflex angle sum must be equal to  $1080^\circ - 180^\circ = 900^\circ$ . This was very satisfying for the teacher as students had used their reasoning to justify a new finding and had taken the lesson to a new level, even though she had not anticipated it or prepared for it.

One of the key features of a DGS is that students learn geometry in an environment where conjectures can be made and quickly examined. Students can use the Algebra and Spreadsheet views to test their conjecture while varying parts of a figure through dragging. A large number of examples can be explored in a relatively small amount of time and this helps to make generalizations regarding properties of geometrical figures. In the simple triangle exploration task described in this article, the students abstracted and generalized a pattern in the angles of a triangle and formed a conjecture regarding the angle sum property. Zoltan Dienes (1963) had explained this process of abstraction and generalization as the key components of mathematical thinking. According to him, mathematical learning is based on the perceptual variability principle as well as the mathematical variability principle:

“The perceptual variability principle stated that to abstract a mathematical structure effectively, one must meet it in a number of different situations to perceive its purely structural properties. The mathematical variability principle stated that as every mathematical concept involved essential variables, all these mathematical variables need to be varied if the full generality of the mathematical concept is to be achieved.” (p.158)

Perceptual variability principle (also referred to as the multiple embodiment principle) suggests that children learn mathematical concepts best when they experience a concept through a variety of physical contexts in the form of concrete materials and manipulatives. Thus, to abstract the concept of the fraction  $\frac{1}{2}$ , various physical embodiments such as circular cut-outs, rectangular rods, beads and counters may be used to represent the fraction in multiple ways. In the context of the triangle exploration by the grade VI children, experiencing the angle sum property by measuring angles of different triangles using a protractor and by using triangular cut-outs, led to perceptual variability of the angle sum concept.

The mathematical variability principle suggests that in order to abstract a concept, even the irrelevant attributes of a concept must be experienced to enable the learner to single out the general mathematical concept. For example, to abstract the concept of a square, the learner must be able to decipher that the equality of the sides and all angles being right angles are relevant invariant properties of a square whereas side length and orientation are not. Thus, when a square is experienced through mathematical variability, one can see squares of different side lengths and orientation, all sharing the common property of the equal sides and right angles. Returning to the context of the triangle exploration, GeoGebra allowed the students to vary the mathematical attributes of the triangle such as side lengths and angles (which are the irrelevant attributes) and observe that the

angle sums (both interior and reflex) remain invariant. This helped them to quickly abstract and generalize the notion that the interior angle sum of a triangle is  $180^\circ$  while the reflex angle sum is  $900^\circ$ . The formal proof of these results are usually addressed at the secondary school stage. However, GeoGebra may be used to help students to visualise these results (and other similar results) through hands-on explorations. Using the approach suggested in this article the reader may try to verify that the sum of the interior angles of a quadrilateral is  $360^\circ$  and the reflex angle sum is  $1080^\circ$  and work towards a generalisation for polygons with  $n$  sides.

Through this article, we have tried to illustrate that the dragging feature of a dynamic geometry software such as GeoGebra, can enable students to explore properties of geometrical shapes. Varying different attributes of a shape brings to the fore its invariant properties amidst variation which, in turn, plays a pivotal role in forming conjectures. Conjecture making is an important aspect of learning geometry, as it is a precursor to argumentation and proof and can be easily accessible to students in a dynamic geometry environment.

### Downloading GeoGebra

In order to download GeoGebra please proceed as follows

**Step 1:** Go to <https://www.geogebra.org/download>

**Step 2:** Click on **App downloads** on the left hand side of the screen

**Step 3:** Scroll down to **GeoGebra Classic 5** on the right hand side of the screen and click on **Download**.

### References

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