review

Book Review:
Three centuries of brain racking discovery

$$x^n + y^n = z^n\ ?$$

Fermat's Enigma – The Epic Quest to Solve The World's Greatest Mathematical Problem

by Simon Singh

Reviewed by Tanuj Shah

To talk about a book on mathematics as 'entertaining' or a 'page-turner' may look out of place; but that is exactly how one would describe Simon Singh's book, *Fermat's Enigma*. The book starts in a dramatic manner: "This was the most important mathematics lecture of the century". Singh is writing about a lecture to be delivered by Andrew Wiles on 23 June, 1993; he was going to sketch a proof of Fermat's last theorem in this lecture. It was known as the 'last' theorem because it was the only remaining 'theorem' stated by the 17th century mathematician Pierre de Fermat which had neither been proved nor disproved, despite close attention given to it over the course of three and a half centuries by some of the greatest mathematicians. (Technically it ought to have been called a 'conjecture' as no proof had been found as yet). One can imagine an atmosphere of tension and excitement in the lecture hall at the prospect of the theorem finally being proved.

What Singh manages to do in the book is weave a story with several strands into a colourful tapestry. The story navigates between biographical, historical and mathematical topics in a fluid and intriguing manner. It captures the spirit that drives and inspires mathematicians to take on intellectual challenges. The protagonist is Andrew Wiles, who as a ten year old dreamed of solving one of the most enduring problems of mathematics – that of finding a proof of Fermat's last theorem, or FLT as it is called – and ultimately went on to solve it.

The FLT states that for the equation $x^n + y^n = z^n$ there are no solutions in positive integers when n is an integer greater than 2.

Singh starts by looking at the origins of the equation in ancient Greece in what we call the Theorem of Pythagoras.[1] This is the case $n = 2$ of the equation, that is, $x^2 + y^2 = z^2$. There is a short biography of Pythagoras, describing how he starts the 'Pythagorean Brotherhood' dedicated to discovering the meaning and purpose of life. He believed that numbers held a special key to unlocking the secrets of the universe. The Brotherhood was fascinated by notions such as *perfect numbers*, i.e., numbers whose proper divisors add up to the number itself (for example, 6). Their world of numbers consisted of the counting numbers and rational numbers, which are ratios of counting numbers. They found a surprising number of connections between these and nature, including the ratios responsible for harmony in music. However their strong belief in the importance of rational numbers proved to undermine further progress by the Brotherhood in the field of Mathematics. There is an apocryphal story where one of the disciples proved that $\sqrt{2}$ is irrational; the Brotherhood felt that this threatened their worldview, which was based on rational numbers, and supposedly the disciple was put to death. However Pythagoras can be credited with laying the foundation of modern mathematics by introducing the notion of proof: starting with a statement that is self evident (an axiom) and arriving at a conclusion through a step-by-step logical argument. The theorem that goes by his name is true for all right angled triangles and it is not necessary to test it on all right angled triangles, as it rests on logic that cannot be refuted. (One of the few hundreds of proofs that exist is given in the appendix of the book). Teaching the Pythagorean Theorem by giving the historical back ground would surely broaden students' horizons and deepen their interest in the topic.

Singh makes a detour at this point, making a distinction between *mathematical proof* and *scientific proof*. The demands made by mathematical proof are absolute; it has to be true for all cases whereas a scientific theory is only a model or an approximation to the truth. This is one reason why the Pythagorean Theorem remains accepted 2500 years after it was first proved, while many scientific theories have been supplanted over the years. These are ideas that a teacher can incorporate while teaching the theorem. Further along, different types of proofs like *proof by contradiction* and *proof by induction* are explained in a lucid manner with examples given in the appendix, which an average 14 or 15 year old would easily be able to follow.

Chapter 2 and 3 focuses on some prominent mathematicians who tried to tackle the problem, starting with Fermat, the person who posed the problem. Fermat was a civil servant – indeed, a judge – who devoted all his leisure time to the study of Mathematics. He was a very private man and hardly met any other mathematician; the only one with whom he collaborated with was Blaise Pascal, on formulating the laws of probability. With Father Marin Mersenne he would share his findings, and Mersenne in turn would pass on the news to other mathematicians. Fermat also had a hand in developing calculus; he was one of the first mathematicians to develop a way of finding tangents to curves. However, the reason he has become a household name is for his 'last' theorem, which he jotted in the margins of the *Arithmetica*, adding: "I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain." This statement spurred a large number of mathematicians to try and prove it, while others contested the claims.

Following this, the author gives a brief biography of Leonhard Euler, one of the greatest mathematicians of the 18^{th} Century. There is enough material here that can be shared with students to give them a glimpse of Euler, and they will certainly enjoy tackling the problem of the Königsberg bridges. There is also a small diversion into the structure of numbers to introduce the idea of imaginary or complex numbers. Euler, who made the first breakthrough on the problem by giving a proof of the case when $n = 3$, had to make use of imaginary numbers. Teachers can test students' understanding of exponents by asking what other cases of the theorem have been proved once you prove it for $n = 3$. (The theorem in this case states that the equation $x^3 + y^3 = z^3$ has no solutions in positive integers.)

The next mathematician to make a breakthrough on this problem was Sophie Germain. The author engagingly brings through the difficulties women had to undergo to establish themselves in this field, which was regarded as a domain for men only. The story of how her father tried to dissuade her from pursuing mathematics by taking away the candles will bring a tear to most. The determination with which she continued her studies including taking up an identity of a man will inspire girls; even now, mathematics tends to be thought of as a subject for boys. It will definitely help in puncturing some stereotypes that people hold.

The second part of the book focuses on the discoveries made in the 20th Century that finally helped in cracking the theorem. Though one may not understand the mathematics behind 'modular forms' or 'elliptic curves', Singh provides good analogies to help the reader keep up with the story without making any excessive technical demands. How two Japanese mathematicians linked the above two areas of mathematics with the Taniyama-Shimura conjecture which led to new approaches in tackling Fermat's last theorem is described in a riveting manner, with a touching account of the tragedy that befell one of them. All along Singh keeps the story moving by giving details of Wiles' career and his attempts at solving the problem, which finally culminates in his lecture in Cambridge in June 1993. However, this is not the end of the story; at the beginning there had been a hint that there was more to come by saying "While a general mood of euphoria filled the Newton Institute, everybody realised that the proof had to be rigorously checked by a team of independent referees. However, as Wiles enjoyed the moment, nobody could have predicted the controversy that would evolve in the months ahead."

Simon Singh has managed to show that mathematicians are people with a great passion to discover the highest truth, and he has certainly succeeded in portraying mathematics as a subject of beauty. The book will inspire teachers and students alike, and is recommended to all classes of readers.

Reference

[1] Editor's note: The article by J Shashidhar, elsewhere in this issue, gives more information on the history of the Pythagorean Theorem.

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