Angle Trisection – An Approximate Procedure

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It is well-known that there is no exact procedure for trisection of arbitrary angles if one uses only the available Euclidean instruments (the straightedge and the compass) and stays within the constraints specified in Euclidean constructions. However, approximate procedures do exist, and the challenge can be to find simple procedures for which the error is very small.

In this article, we present one such procedure.

Motivation for the procedure

The idea behind the procedure emerges from the following observation.

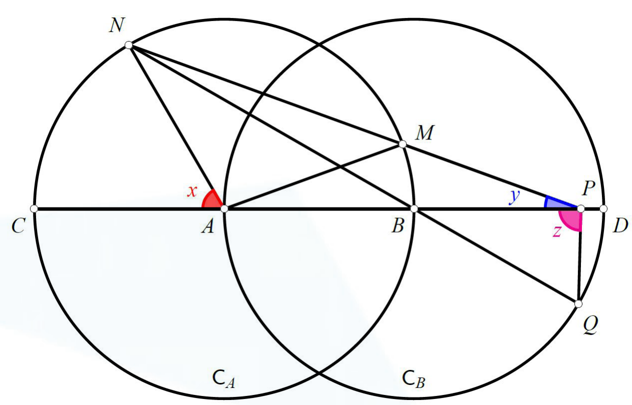

Figure 1

- Given any two points A and B. Let the distance AB be referred to as 1 unit.
- Draw circles C_A and C_B centred at A and B respectively, with radius 1 unit each (Figure 1).

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- Join AB and extend it to intersect C_A and C_B again at points C and D, as shown.
- Let P be any point on line AB, between B and D.
- Draw a circle C_P centred at P, with radius 1 unit. Let C_P intersect C_B at M (and one other point which we do not name as it is not needed).
- Join PM and extend it till it meets C_A again at point N. Join NA. Let $x = \measuredangle NAC$, $y = \measuredangle NPA$.
- Join NB and extend it till it meets C_B at point Q. Join PQ.
- It is now easy to verify that $\measuredangle NAC = 3\measuredangle NPA$, i.e., $y = x/3$. This relation is exact.
- If we try out this construction and measure the size of $\measuredangle BPQ$, we notice that *the angle is almost a right angle. Moreover, this remains true regardless of where P is located between B and D.*

It is the observation made at the end that gives rise to an approximate trisection procedure.

Proposed approximate procedure for trisection of angle

As noted above, the fact that $\measuredangle BPQ$ is almost a right angle plays a crucial role in the procedure. Here are the actual steps (see Figure 2).

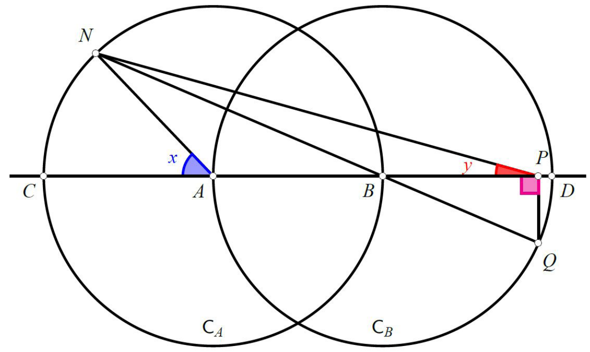
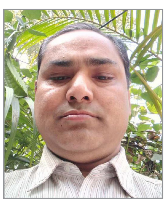


Figure 2

- Let the angle to be trisected be denoted by x.
- Mark any two points A and B, 1 unit apart. Draw circles C_A and C_B centred at A and B respectively, with radius 1 unit each. Join AB and extend it to intersect C_A and C_B again at points C and D.
- Locate a point N on C_A such that $\measuredangle NAC$ is equal to x (the angle to be trisected).
- Join NB and extend it beyond B till it meets C_B again at point Q.
- Draw QP perpendicular to line CD (with P on CD). Join NP.
- Then it will be found that $\measuredangle NPC \approx 1/3\ \measuredangle NAC$, i.e., $y \approx x/3$.

The reader could try out the procedure and check the closeness of the approximation.



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