## Maths Club

# A fascination for counting

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We have been running a Maths Club in our school for a number of years, for class 7 students. We meet for one hour every week. We find that at this age, students have a need to explore the subject at a greater depth and a great desire to venture out and make connections with real life applications. They are also able to appreciate the aesthetic aspects of the subject at this age. The club is open to everyone irrespective of their mathematical ability. The aim of the Maths Club is to open up the ways in which students perceive Mathematics; to help them see the beauty and power of the subject. One topic that never fails to fascinate the children is that of counting. It is accessible for children of all abilities, and reveals patterns very quickly.

It is important to provide interesting narratives and contexts. We start off by talking about how blind people are able to read. Somebody suggests that the letters could be raised on a paper. Somebody disagrees, saying it is not a very efficient way of reading and would take a long time if the blind had to feel each letter. At this point somebody interjects and says that they have seen Braille books, which consists only of dots. The children are told that each character in Braille is represented within a 2×3 rectangle by raising dots in a particular pattern. There is only a single way in which each character can be represented. For example the letter 'm' would be represented as in Figure 1.

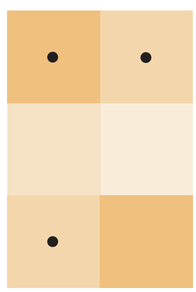

Figure 1

Now the children are given the task of finding out how many different characters could be represented in this way on a 2×3 rectangular grid, i.e., how many different patterns of dots and blanks are there. After some time we gather together and discuss how everyone went about the task and the difficulties that they encountered. It is observed that there are a large number of possibilities and it may not be easy to verify if we have covered them all. Somebody suggests that it may be worth doing things systematically: let's look at the possibilities of 0 dots, 1 dot, 2 dots etc. separately and then add them all up. There is only one way of representing the rectangle with zero dots. For 1 dot we can see that it can be put in any of the 6 spaces in the rectangle, so therefore there are 6 possibilities. For 2 dots one needs to go about it systematically by keeping one dot fixed in each of the 6 spaces and then adding the $2^{nd}$ dot and being careful that there is no double counting. Here we find 15 possibilities (through systematic counting we can see that it is 5+4+3+2+1). As we write down the possibilities children start noticing an interesting pattern with reflective symmetry. Somebody suggests that the case for 4 dots is the mirror image of 2 dots, only that the dots and empty spaces are reversed. We display the results in a table:

| Number of dots | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Number of ways | 1 | 6 | 15 | 20 | 15 | 6 | 1 |

The final sequence looks like this:
1, 6, 15, 20, 15, 6, 1. The total number of possibilities turns out to be 64, though somebody suggests that we should ignore the box with zero dots, as this would be confusing for blind people. So we settle on 63 possibilities. A question is posed, whether Braille can work for other kind of rectangles apart from 2×3. We agree to explore this question in the following week.

When we meet again, they are shown a blank flag with four stripes. Then the question, how many different ways can we colour the flag if we are only allowed to use the colours black and red, is posed (it is made clear that none of the stripes can be left blank). We gather together after five minutes and discuss how every one is going about the task. Quite a few remember the previous week's suggestion of doing it systematically, starting with one colour i.e. how many flags with 0 red stripes, 1 red stripe, 2 red stripes, etc. They are then given some more time to work it out. Those who finish quickly are given the task of exploring a flag with 5 stripes. As we get together we observe that for the 4 striped flag we get a reflective pattern: 1, 4, 6, 4, 1, and the total number of possibilities is 16. Somebody notices that this looks very similar to last week's question. We see that in both the situations there were 2 possibilities —a dot or a blank in the first one, and red and black in the second. Discovering this has a powerful effect on the children since situations with two possibilities can now be modelled in this manner. The question raised at the end of the first session can now be answered. We could use different sized rectangles for writing Braille; however we would not be able to cover all the characters with a rectangle smaller than 2×3. Those who attempted the 5-stripe problem tell us that there are 32 possibilities.

However it would not be enough for the alphabet and all the punctuations to be shown. We decide to put all our findings down and see if we can spot any more patterns.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Four-stripe problem | 1 | 4 | 6 | 4 | 1 | | |
| Five-stripe problem | 1 | 5 | 10 | 10 | 5 | 1 | |
| Six-stripe problem | 1 | 6 | 15 | 20 | 15 | 6 | 1 |

Most are quick at recognising the Pascal's triangle, which they have been exposed to earlier. There is still more interest in this topic and so there is a promise of more to follow.

In the third session, we explore the number of paths that can be taken from one end of a rectangular grid to another, without backtracking. We start in one corner cell (S) and move either down or right until we reach the corner cell that is diagonally across (E). At each step one has a choice of going down or to the right (see Figure 2). How many possible paths are there?

| | | |
|---|---|---|
| Start | | |
| | | |
| | | End |

Figure 2

An interesting pattern emerges if one starts writing down the number of ways to each cell in the grid as shown in Figure 3.

| | | | |
|---|---|---|---|
| Start | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 |
| 1 | 3 | 6 | 10 |
| 1 | 4 | 10 | 20 |

Figure 3

The children are quick to spot Pascal's triangle yet again, and fascinated at the way it turns up in such unexpected ways. Later when children attempt expanding binomial expressions with different indices, they will have great satisfaction in spotting these patterns again. If time permits, then one could explore the problem of tossing different number of coins and looking at the outcome of heads and tails.

This series of three lessons would have given them pleasure in spotting patterns and developed their abilities to spot and deal with similar situations in counting.

While working on the Braille problem there are lots of interesting asides one can talk about depending on the interest level. One can talk about the biographical account of Louise Braille, mention that earlier the rectangular grid used to be larger and this made it difficult to read, until Louise Braille introduced the standard 2×3 grid which is now used. There is also a separate language for representing Mathematics and reading a music score for the blind.

TANUJ SHAH teaches Mathematics in Rishi Valley School. He has a deep passion for making mathematics accessible and interesting for all and has developed hands-on self learning modules for the Junior School. Tanuj Shah did his teacher training at Nottingham University and taught in various schools in England before joining Rishi Valley School. He may be contacted at tanuj@rishivalley.org.