At Right Angles
A Resource for School Mathematics

COORDINATES

PADMAPRIYA SHIRALI

Azim Premji University

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Rishi Valley

COORDINATES

I currently teach coordinate geometry in Class 9 (ICSE Curriculum) and it is not clear to me why the basics of coordinate geometry are not included in the school syllabus (in India) in primary and upper primary levels. My experience of using it with young students has been enjoyable. Coordinate geometry can be introduced at an early stage, in both the primary and upper primary levels.

The basic concept can be appreciated by young children and the topic has plenty of scope for generating geometric exploration. It is a topic that lends itself to active participation and the usage of pegboards and geometric software. Mathematical games incorporating coordinates are great fun and develop strategic thinking.

Coordinate geometry at the middle school level provides a connection between algebra and geometry through its usage of lines. It helps students visualise simultaneous linear equations in a graphic form.

These coordinates are Cartesian coordinates and are also referred to as rectangular coordinates. The positions in the Cartesian system are defined by the distances of the points from the two axes. The scales used are linear.

There are also other types of coordinates used in maps, polar coordinates and 3-dimensional coordinates. In this article, we confine ourselves to Cartesian coordinates.

Coordinates can be introduced at the primary level with positive numbers and a transition to the usage of negative integers can be made at the upper primary level.

At the primary level, coordinates are used to describe the locations of objects using the language of position, i.e., across from the left (column) and then up or down (row). Rows and columns – words that are familiar to students – are the starting point. In an ordered pair, e.g., (3, 2), the convention is to state the column (horizontal reference) first and the row (vertical reference) second.

In an array arrangement, the coordinates refer to a discrete object. On a square grid, the location is a specific point at the intersection of the lines.

Students are generally made to stand in rows and columns either during assembly or games time. Indoors, in classrooms, they are generally seated in rows and columns. This array arrangement lends itself very well to locate each student using row and column numbers. While introducing the topic teachers must ensure that the students are familiar with the words row and column.

Note: There are some differences in the way we begin to count rows. In a physical arrangement, as in a theatre the first row is in the front. In a table format it begins from the top row, i.e., we start at the top and work our way down (top row is row 1). In the coordinate system we start at the x-axis (row 0) and work our way up. Numbering for the columns remains the same in all the situations. This need not pose a problem to understand naming of locations as the class arrangement corresponds to the graphing system.

ACTIVITY 1: POSITION, PLEASE!

Objective: Give and follow instructions using the language of position (coordinates)

Figure 1

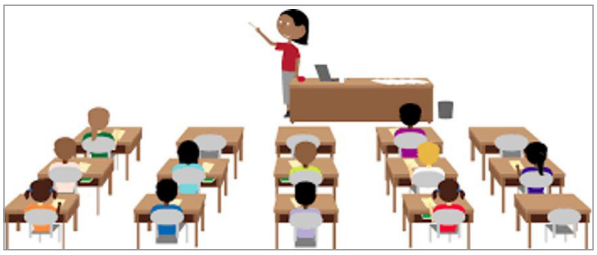

Figure 2

The teacher points to the student in the first row and the first column and gives him/her the location number (1,1) and explains the ordered pair as "He/she is in the first column and the first row, so the location number is (1,1)." The teacher continues to give location numbers for others in row 1 by going in order: (2,1), (3,1), … (6,1), explaining each time the reason for the numbering process. After finishing with the first row, the teacher draws the attention of the students to the second row and starts with (1,2) by emphasising the words first column, second row, (2,2), etc.

The teacher must emphasise that the first number in the ordered pair refers to the column number, and the second number in the pair refers to the row number. Use the words row and column repeatedly till the students can comfortably associate the first number of the ordered pair with the column and the second number of the ordered pair with the row.

Practice

Once the students are clear about their location numbers (coordinates) the teacher can call out the name of a student, he/she has to give their location.

The teacher can call out a coordinate pair and the student in that position has to respond with an appropriate action like standing up.

Further interesting questions that can be asked are:

"Can all those whose row number equals column number stand up?" What will be the result?

"Can all those whose row number is less than the column number stand up?" What will be the result?

"Can all those whose numbers add up to 7 stand up?" Do students notice any symmetry here?

ACTIVITY 2: DOT PUZZLE

Objective: Use ordered coordinates to plot and connect

A coordinate dot puzzle consists of a set of given coordinates that students plot on a square dot/ grid paper. They plot and join the points in the order that they are given. The figure can be a cartoon character or any other picture that children can recognise.

They could also create problems of this kind with their designs, note the coordinates in order and share it with others.

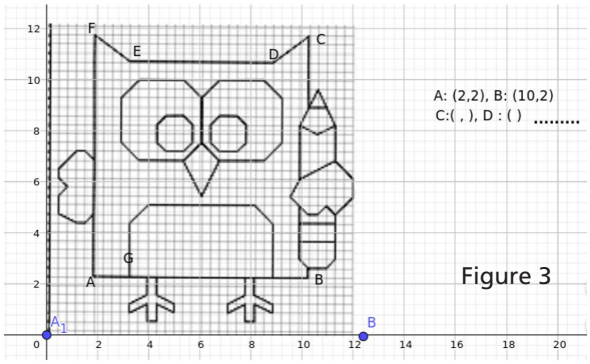

Figure 3

ACTIVITY 3: TREASURE MAP

Objective: Use coordinates to locate treasure on a map superimposed by a grid. (GeoGebra can be used to prepare such pictures)

Pose questions: 'Where is the casket?' 'Where is the boat?' And so on.

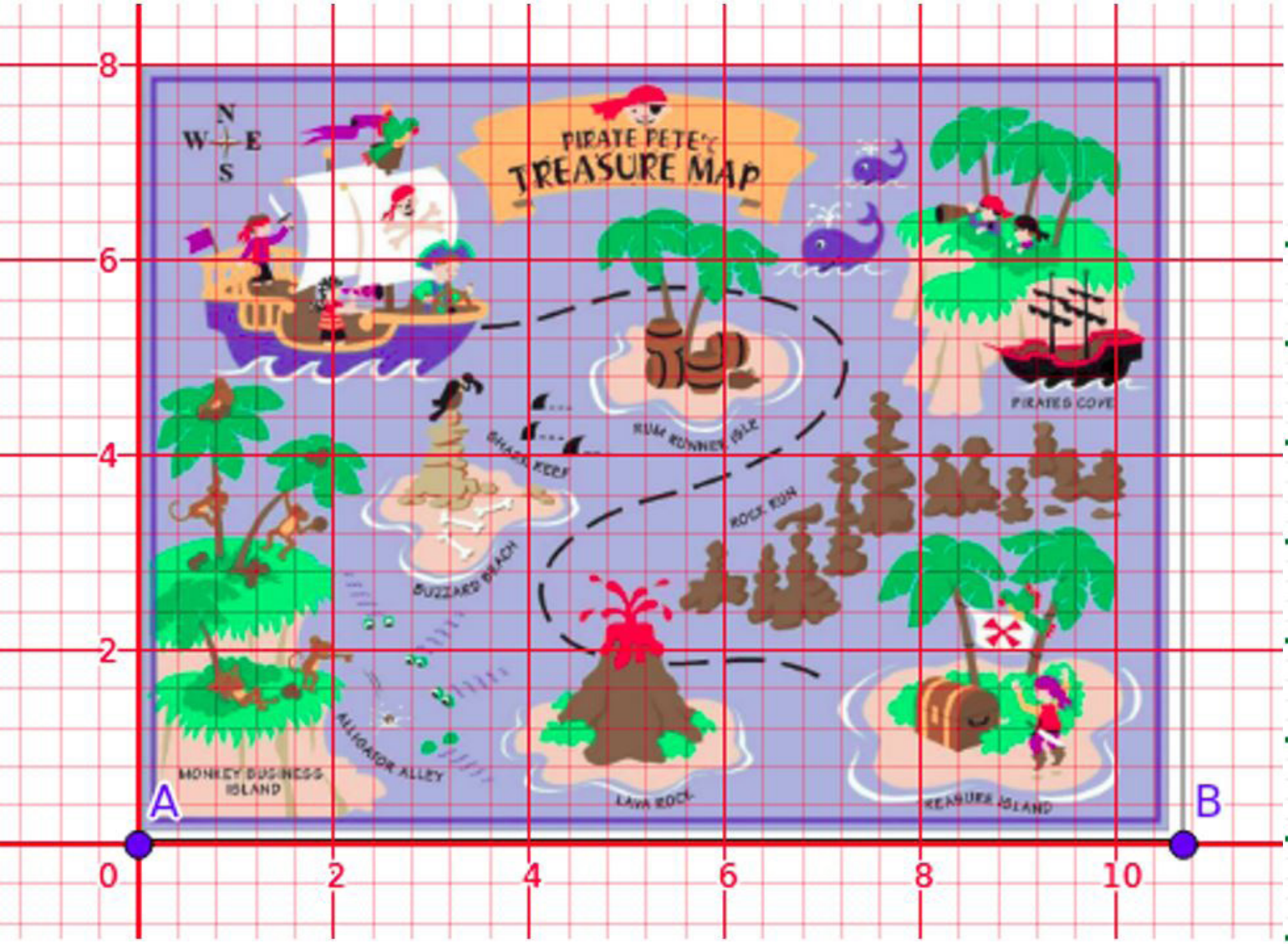

Figure 4

ACTIVITY 4: OUTDOOR TREASURE HUNT

Objective: Maps and usage of coordinates

Treasure hunts produce a lot of excitement and are generally organised using clues involving puns. It is interesting to modify the game and use coordinates. Teachers can use the school playground map with a coordinate system drawn over it. Each team will need copies of this map with the superimposed coordinate system.

Students can be separated into two teams. One team can bury some treasure at four different places on the school playground and plot the location of these places on the school map. They can then provide the coordinates of these locations to the other team. The second team has to locate the treasure based on the set of coordinates that have been given to them.

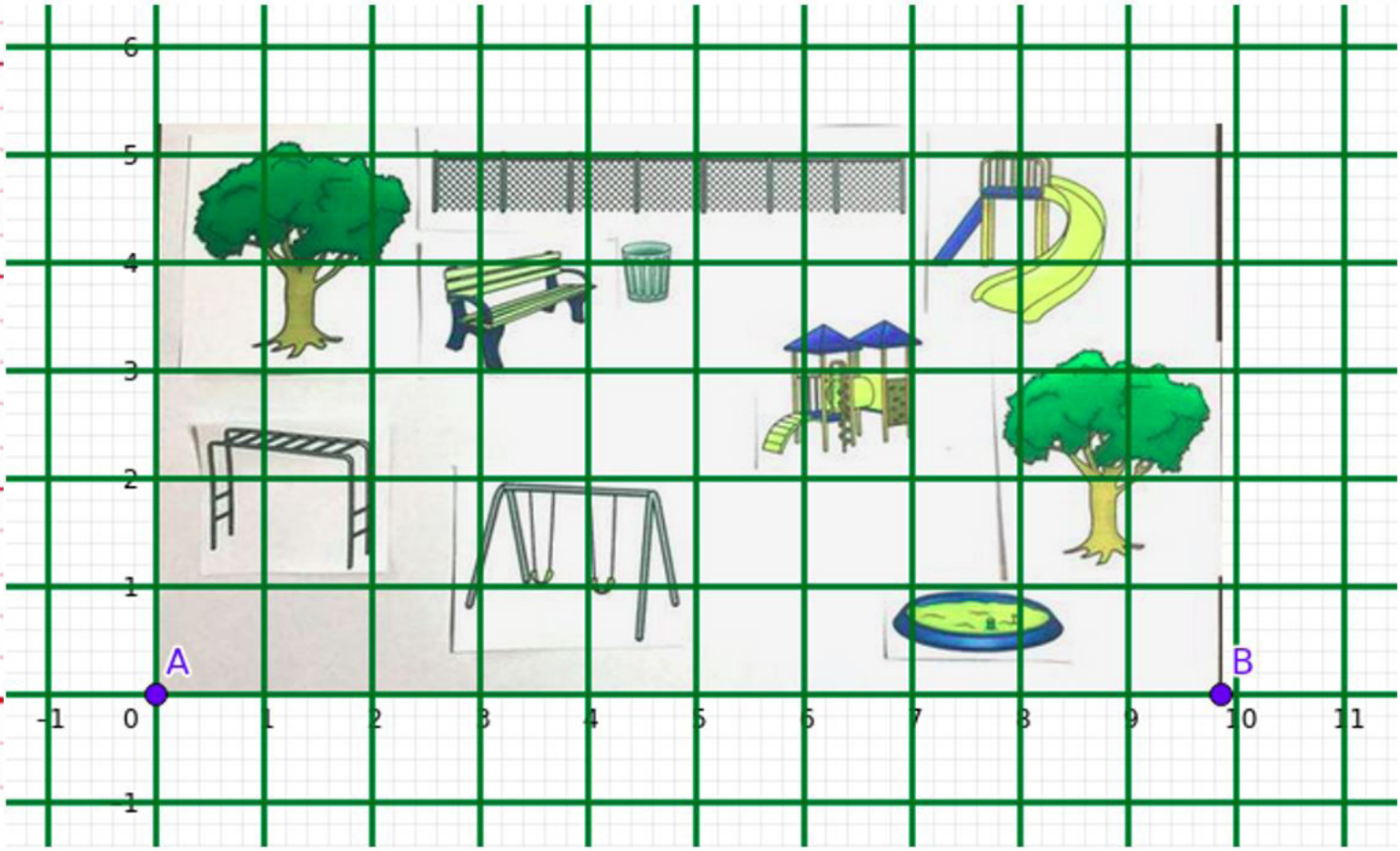

Figure 5

ACTIVITY 5: DOTS AND LINES

Objective: Plot points on a square dot paper and notice the line shape that emerges.

Students can use different colours for each question or create multiple grids on a single square sheet of paper and use one grid for each question.

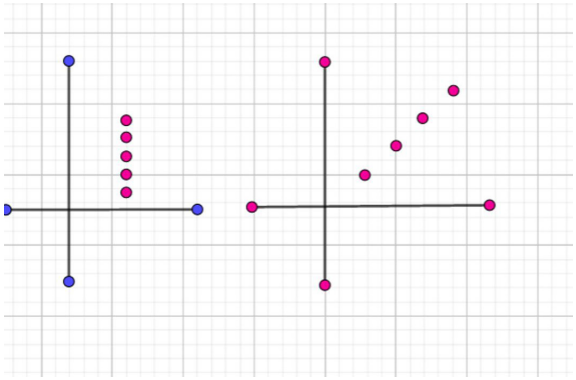

Figure 6

Questions:

Plot all the locations which have 4 as their column number. What do the points look like?

Plot all the locations which have 3 as their row number. What do the points in the figure form?

Plot all points where the row number equals the column number. What kind of a line is this? (Students may not know the word *diagonal*, so they may use their own words to describe it.)

Plot all points whose sum adds up to 8. What do you notice?

ACTIVITY 6: FIND MY ALPHABET

Objective: Visualisation, practice with coordinates and deduction of the alphabet

This activity can be done between two students. One student makes a letter of the alphabet (the digital form) on the grid and shares one coordinate pair at a time with the other student. Based on the information the other student has to deduce the letter. The second student must try to figure it out before all the coordinate pairs are given.

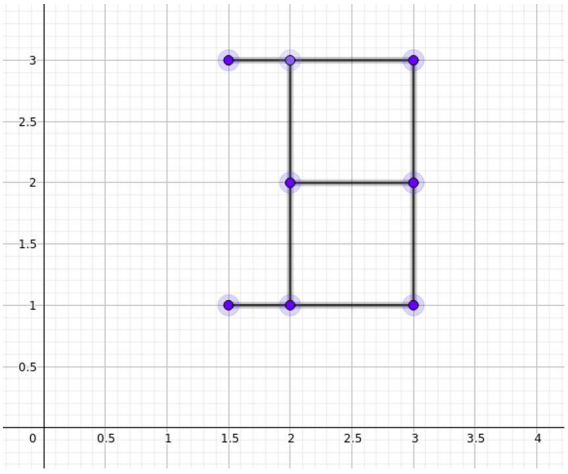

Figure 7

ACTIVITY 7: DOTS AND PATTERNS

Objective: Predicting the coordinates of a patterned figure

Students note down the coordinates of a given diagram in a table format and predict the next two coordinates.

	row	Column
A	0	3
B	1	4
C	2	5
D	3	6
E	4	7

Figure 8

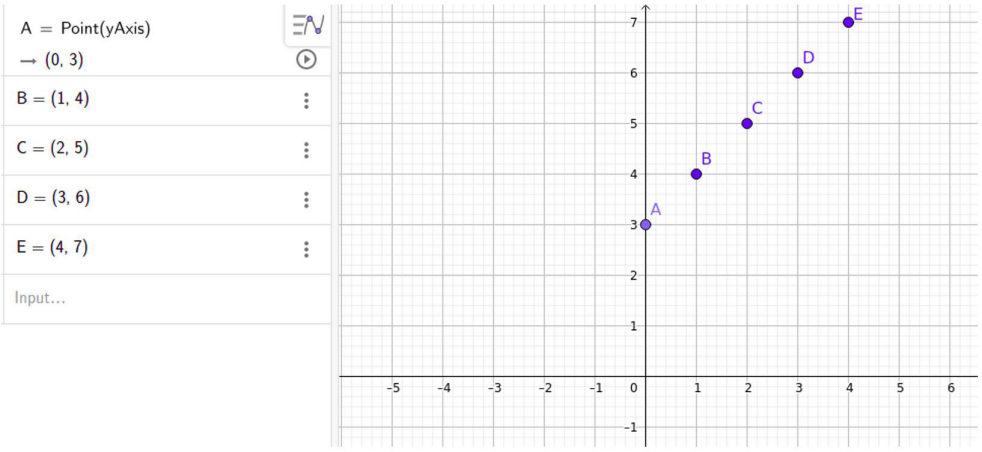

Figure 9

Here is one more example of a figure for prediction of the next two points.

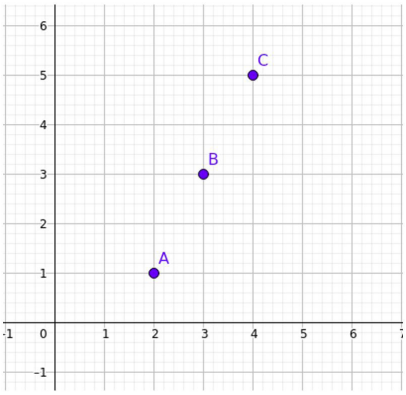

Figure 10

Note: At the upper primary level, teachers can begin to refer to column and row numbers as *X* coordinate and *Y* coordinate. Also, introduce quadrants and let students note the signs of numbers in each quadrant.

ACTIVITY 8: DOTS, LINES (FOR UPPER PRIMARY)

Objective: Plot points on a square dot paper and notice the lines or regions that emerge.

Plot all locations which have -4 as their row number. Plot all the locations which have -1 as their row number. In what way is this the same as the previous figure? In what way is it different?

Connect two points A (4,0) and B (-2,0). What can you say about the column number of all other points on this line? What is staying the same (constant)? What is changing (variable)?

Can you answer this without plotting the points?

If the two points (2, 1) and (6, 1) are joined, will they make a vertical line? Or will they make a horizontal line? How did you make out?

Where do you think the midpoint of this line segment will lie? How did you find out?

Now check your answers by plotting them.

ACTIVITY 9: GRIDLOCK

Objective: Strategic thinking

Materials: 6 x 6 Grid, two dice, 10 counters of two colours

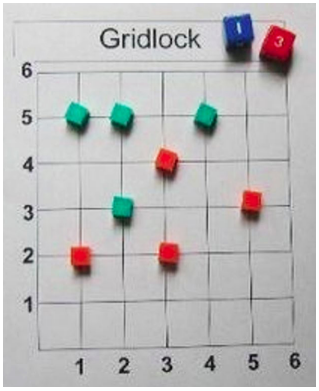

Figure 11

The game can be played by two players at a time. Each player casts the two dice and places his/ her counter on the grid. Ex. If the dice shows 2 and 3, the player can choose to place the counter either on (2,3) or (3,2) positions. Both the players take turns to place their counters on the grid. If the position is already occupied, the player skips his/her turn. Each one tries to get a series of 4 counters in a line to win the game.

ACTIVITY 10: REGIONS

Objective: To show inequalities

Use a 4 x 4 grid. Plot the points where the row number is less than or equal to 2 and shade the region. How will you describe the picture?

Plot the points where the row number is less than the column number in the first quadrant. Now plot points following the same rule in the second quadrant. What do you notice? Shade these regions.

Now plot the points observing the same rule in the third and fourth quadrants. What happens?

What would happen if you did the reverse, i.e., the column number is less than or equal to the row number?

In Figure 12 observe each quadrant separately to state what is happening out there.

How are the coordinates of the points lying on the separating line related to each other?

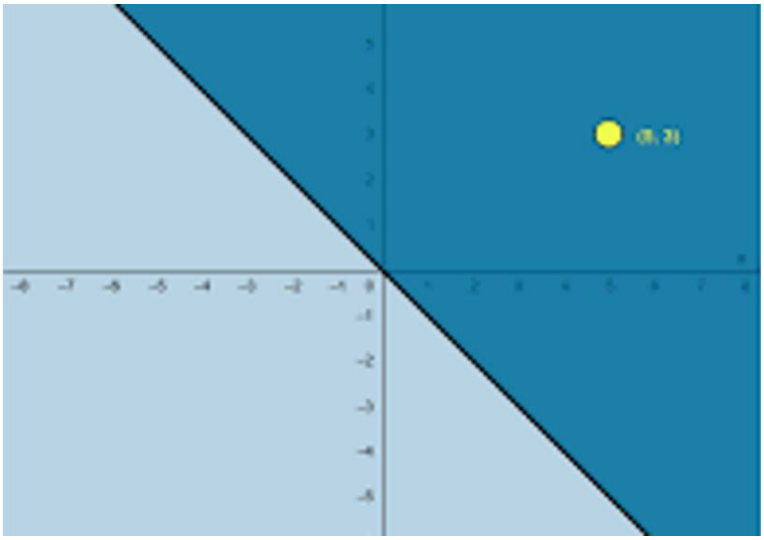

Figure 12

ACTIVITY 11: HIT OR MISS GAME!

Objective: Give and interpret instructions using coordinates

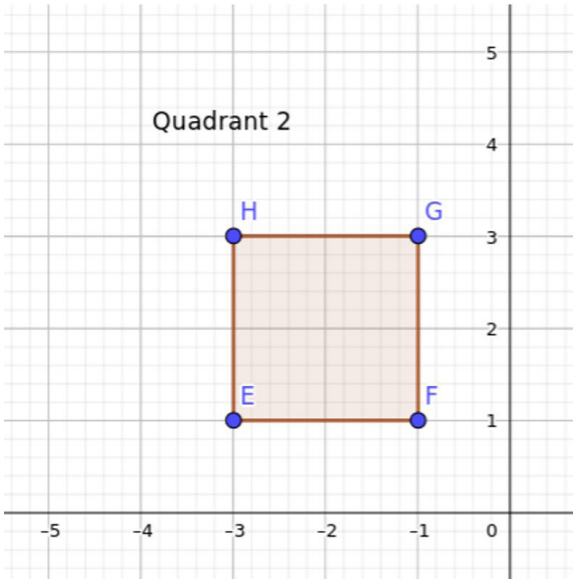

Figure 13

This can be played by two students. Players use a grid paper of 5 x 5 size. One player shades a 2 x 2 square box in his/her grid paper. It is hidden from the second player. They need to share where the shape lies, in which quadrant. The second player calls out a coordinate pair and the first player responds by saying hit or miss. If the coordinates called out are part of the shaded box it is a hit, if not it is a miss. Based on the response the second player will call out another set of coordinates.

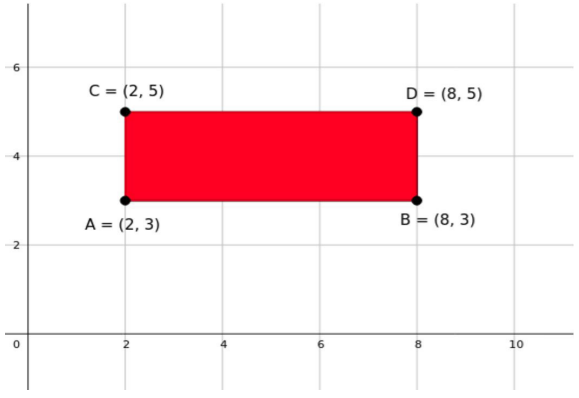

Figure 14

Players are given a maximum number of 12 chances to call out. If all the coordinates of the corners of the box are found then the player has won.

The game can be modified to find a rectangle but the level of challenge rises.

ACTIVITY 12: HIDDEN TREASURE

Objective: Visualisation
Materials: 6 x 6 square grid

The students can be made into two teams. The first team decides on a set of coordinates as the location for a treasure on a 6 x 6 grid.

The second team makes an initial guess of the location and gives a coordinate pair and plots the pair on their grid. Based on the guess the first team gives the number of steps required to get to the treasure. The number of steps is obtained by using the shortest route along the horizontal and the vertical lines (not diagonal) from the guessed location to the treasure. The second team notes down the information on their grid as shown in Figure 15. (Ex. A is 5 steps away from the treasure.) They use this information to make their second guess. At each step, the first team gives them the number of steps from the given coordinates to the treasure. This process continues till the second team manages to locate the treasure. The goal is to find the treasure in as few guesses as possible.

Where is the treasure in the example given below?

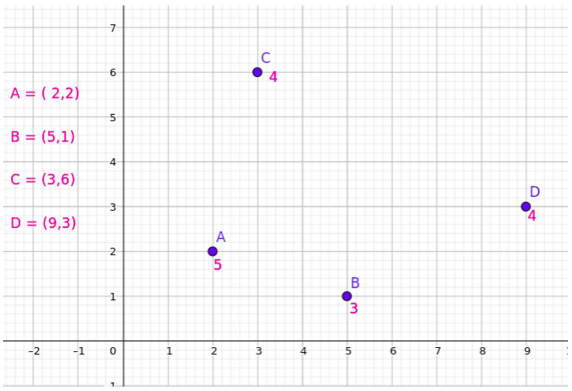

Figure 15

ACTIVITY 13: LINES

Objective: Figuring lines
Materials: 10 x 10 square grid

The line AC continues downwards to a point B. If AC is one-third of the line segment AB, what are the coordinates of B?

If AC is one-fourth of the line segment AB, what are the coordinates of B?

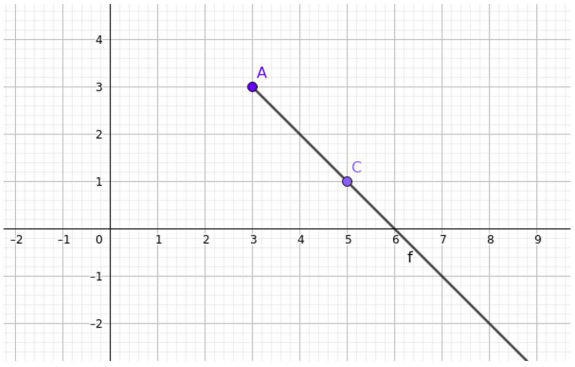

Figure 16

The line segment AB continues in both directions ending at points O and C. If AB is the middle one of three equal segments (one third of OC) what are the coordinates of O and C?

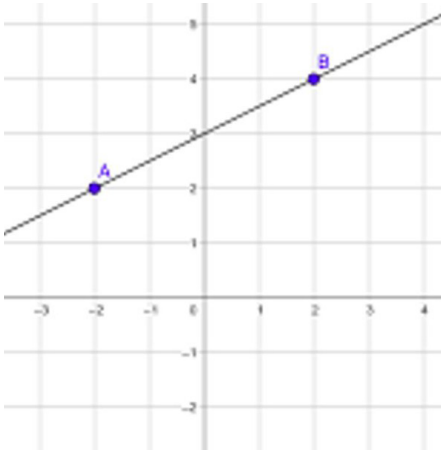

Figure 17

ACTIVITY 14: REFLECTING POINTS, LINES AND SHAPES

Objective: To reflect lines in both the axes
Materials: 10 x 10 square grid

Students can be initially asked to draw the reflections of given points along X-axis and along Y-axis.

As a second step they can reflect lines along x-axis and along y-axis.

They could also do shape reflection to generate symmetric designs.

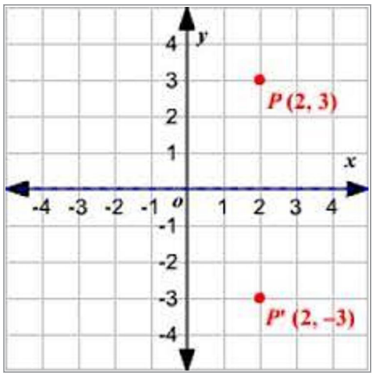

Figure 18

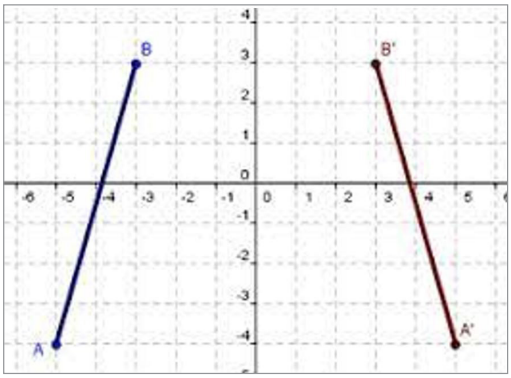

Figure 19

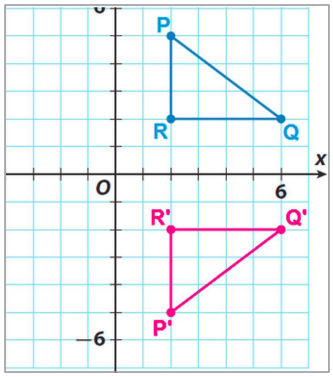

Figure 20

ACTIVITY 15

Objective: Deduce shape
Materials: 10 x 10 square grid

What would be the coordinates of the fourth vertex of this square?

Where can the coordinates lie if it were a kite? What are the possible answers?

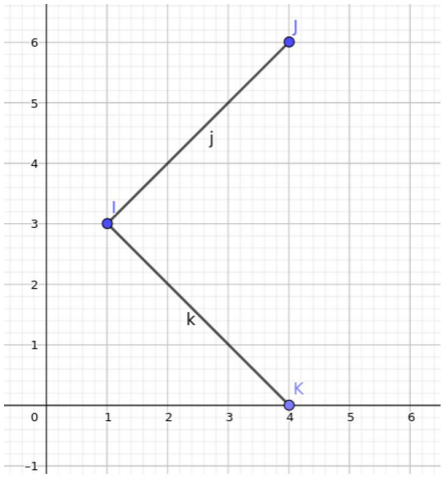

Figure 21

The centre of this rectangle lies at (-1,-0.5). Where do the vertices C and D lie?

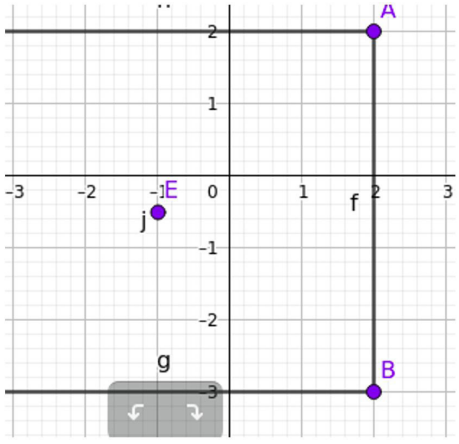

Figure 22

What shape would you get if you were to join the three sets of points (2,0), (8,0) and (5, 2)?

ACTIVITY 16

Objective: Finding patterns in coordinates to predict the next pair.
Materials: 10 x 10 square grid

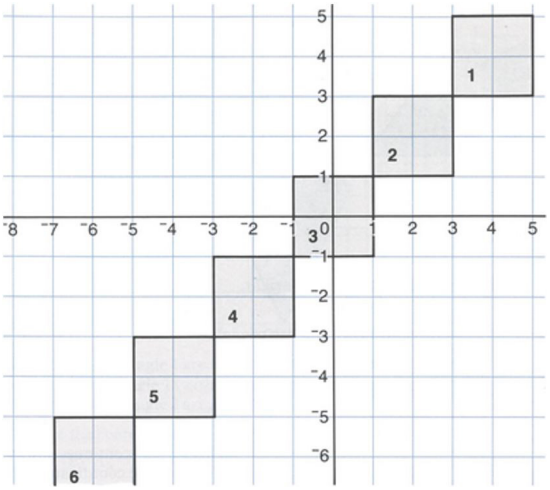

Figure 23

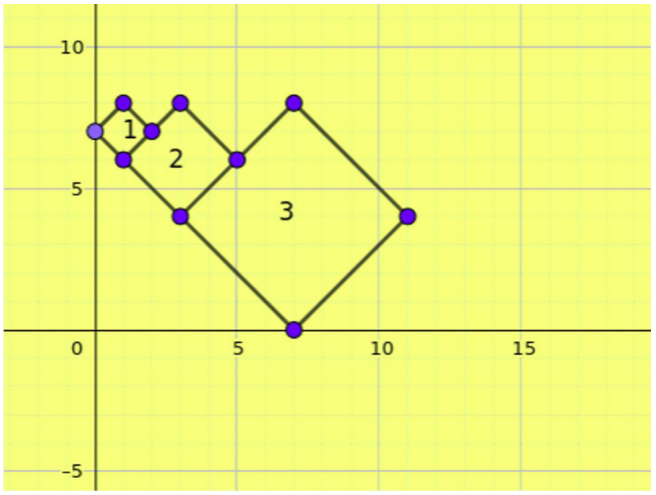

Figure 24

Here is a set of numbered squares in Figure 23 going downwards and to the left.

What will be the coordinates of the centre of the tenth square in Figure 23?

What are the coordinates of the centre of each square in Figure 24?

(The shapes, angled at 45 degrees, are growing downwards in increasing size.)

Here is a problem taken from the Nrich website. See Figure 25.

Where will the vertices of triangle 23 be?

Suggest a quick way of working out the coordinates of any triangle in this figure.

Will your method work if the triangles extended to the left?

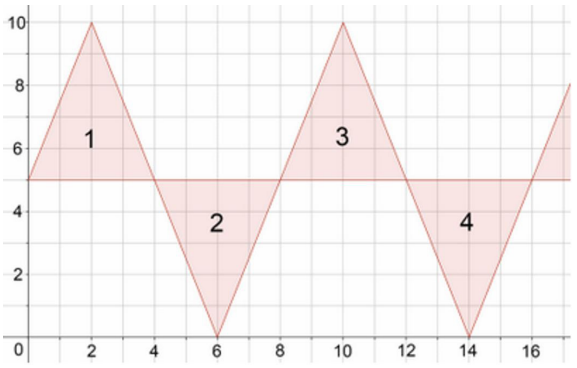

Figure 25

Acknowledgements

SMP
Nrich

PADMAPRIYA SHIRALI

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multi-grade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box.' Padmapriya may be contacted at padmapriya.shirali@gmail.com.