review

Review of

"The Crest of the Peacock: Non-European Roots of Mathematics"

By George Gheverghese Joseph

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Background

If we made contact with an alien civilization, what would we communicate with them, and how? Lacking a common language, how can we expect to make sense at all? Many thinkers have pondered this and among all their divergent views, one thread is common: the language of mathematics presents the best possibility for such communication. For instance, sending the sequence of primes expressed in base one (unary) may be a good idea.

What is the conviction that underlies such a suggestion? We believe that mathematical pursuit of some kind is generic to intelligence and sapience, therefore we expect and look for mathematical pursuits in all civilizations. But then, each civilization and cultural tradition may develop mathematics independently and in its own way, so while communication of what is basic and fundamental may yet be possible, that of sophisticated technique may be hard.

This is an easy argument to accept, and yet hard to internalize in a way that informs our practice, especially of the teaching of mathematics. We have a picture of mathematics as a modern discipline, and trace its roots to ancient Greece and the European Renaissance, absorbing the vacuum in between, but fail to ask what other trajectories of mathematical development might have taken place in other cultures, especially during these "dark ages". The 'we' here includes historians and practitioners of mathematics, all over the world.

Such ignorance and intellectual laziness has come under question in the last couple of decades, and at least in the corners inhabited by historians of mathematics, the Euro-centric account is being interrogated now. The book under review, first published in 1991, contributed in no small way to this change of perspective and was tremendously influential in causing the shift. By now it has seen three editions and is yet relevant, especially in the Indian context, where it has become fashionable to talk of deep mathematical knowledge in 'ancient India' without factual basis. Keeping a clear view of history while yet not buying in entirely to the Euro-centric account, is important for opening our minds to historical progressions of ideas, an integral component of learning.

The central aims of the book (from Chapter 1) are, to highlight:

1. "the global nature of mathematical pursuits of one kind or another,
2. the possibility of independent mathematical development within each cultural tradition followed or not followed by cross-fertilization,
3. the crucial importance of diverse transmissions of mathematics across cultures, culminating in the creation of the unified discipline of modern mathematics".

George Joseph does a remarkable job of addressing these aims, and takes us through a fascinating journey of mathematics in non-European cultures, principally the Asian ones.

Content

The book opens with a discussion of the Euro-centric picture dominant in the history of mathematics, its critique and the necessity to view mathematical development in many cultures of the world, and transmissions between them. Then the author gives a brief account of pre-historic mathematics, such as those found in the *Ishinga Bone*, the mathematics of the Incas and the Mayans, and the development of number systems. (Interestingly, some authors have raised serious questions on mathematical inference from Ishinga bones, Asolom's wolf bones and such. See *The fables of Ishango, or the irresistible temptation of mathematical fiction*, Olivier Keller, Préhistoire de l'arithmetique, Feb 2015.) An important discussion here is on mathematics in Africa, especially geometric designs: very brief, but pointing to an area not generally discussed.

Chapters 3 and 4 discuss mathematics from Egypt and Mesopotamia and Chapter 5 is an 'assessment' of these two. Joseph presents an illuminating picture of the empiricist and algebraic tradition prevalent in Egypt and Babylonia. Since the Greeks had extensive interaction with these societies, Joseph makes a case for how a synthesis of the deductive and geometric tradition of Greece with this algebraic approach might have led to the powerful mathematics that emerged, especially in the works of Archimedes, Ptolemy and Diophantus.

Many of the examples presented of Egyptian and Babylonian mathematics have their origins in people's work, on the empirical need for calculation. For instance, calculate the number of persons needed to move an obelisk. The modeling needs of such tasks and their subsequent abstraction, seems to have led to interesting mathematical constructions. Studying these can be inspiring for today's students, relating to similar tasks in today's world. The tasks are largely arithmetical and measurement oriented, and involving basic algebra, all accessible to a child in middle school. There are also algorithms from Mesopotamia like the one for extraction of square roots, but it is not clear how different and enriching they are.

For shock value, consider the following problem (Example 4.5, Chapter 4): *Calculate how long it would take for a certain amount of money to double if it has been loaned at a compound annual rate of* 20%. You expect to see this in current day high school texts. This is from the Louvre tablets of the Old Babylonian Period, approximately 1500 BC. Here is another, from the Susa tablets of the Old Babylonian Period (Example 4.11, Chapter 4): *Find the circumradius of a triangle whose sides are* 50, 50, *and* 60. It is this problem that leads Joseph to assert: "there can be little doubt that the Mesopotamians knew and used the Pythagorean theorem." Be that as it may, it would be instructive

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TABLE 6.1: MAJOR CHINESE MATHEMATICAL SOURCES UP TO THE SEVENTEENTH CENTURY

Title	*Author*	*Date*	*Notable subjects covered*
Zhou Bi Suan Jing (The Mathematical Classic of the Gnomon and the Circular Paths of Heaven)	Unknown	c. 500–200 BC	Pythagorean theorem; simple rules of fractions and arithmetic operations
Suanshu Shu (A Book on Arithmetic)	Unknown	300–150 BC	Operations with fractions; areas of rectangular fields; fair taxes
**Jiu Zhang Suan Shu* (Nine Chapters on Mathematical Arts)	Unknown	300 BC–AD 200	Root extraction; ratios (including the rule of three and the rule of false position); solution of simultaneous equations; areas and volumes of various geometrical figures and solids; right-angled triangles
Ta Tai Li Chi (Records of Rites Compiled by Tai the Elder)	Unknown	AD 80	Magic square order of 3
Commentary on *Jiu Zhang*	Chang Heng	130	π = square root of 10
Shu Shu Chi Yi (Manual on the Traditions of the Mathematical Arts)	Xu Yue	c. 200	Theory of large numbers; magic squares; first mention of the abacus
Commentary on *Zhou Bi*	Zhao Zhujing	c. 200–300	Solution of quadradic equations of the type $x^2 + ax = b^2$
Hai Dao Suan Jing (Sea Island Mathematical Manual)	Liu Hui	263	Extentions of problems in geometry and algebra from the *Nine Chapters*
Sun Zu Suan Jing (Master Sun's Mathematical Manual)	Sun Zu	400	A problem in indeterminate analysis; square root extraction; operations with rod numerals

continued

Figure 1.

and thought provoking for our students to solve non-trivial problems posed more than 3000 years ago.

Chapters 6 and 7 are on Chinese mathematics. It is very likely that most of our teachers are unaware of the long history of mathematical development that our neighbours had, and of the multiple transmissions between our cultures. Rather than list the many interesting topics, I have reproduced part of the chronology presented by Joseph (Table 6.1 of *Peacock*) in Figure 1.

Chapter 7 is devoted to a specific period, the late 13th and early 14th centuries, during the Song dynasty. Very fine mathematicians such as Qin Jiushao, Li Ye, Yang Hui, and Zhu Shijie lived in this period, and several schools of mathematics flourished. Joseph categorizes the essentially algebraic work of this era into three kinds: numerical equations of higher order, Pascal's triangle (note the period, for what was named after the 17th century French mathematician Blaise Pascal) and indeterminate analysis (solving a system of n equations with more than n unknowns).

We have all heard of the Chinese remainder theorem. Chapter 7 gives a very nice account of the historical development of ideas related to this (apart from geometry in Chinese mathematics). To trigger thought, consider the following problem from the 4th century mathematical text Sun Zu Suan Jing. *There are an unknown number of objects. When counted in threes, the remainder is* 2; *when counted in fives, the remainder is* 3; *and when counted in sevens, the remainder is* 2. *How many objects are there?* In modern notation,

$$N = 3x + 2,\ N = 5y + 3,\ N = 7z + 2,$$

or better,

$$N \equiv 2 \pmod 3,\ \ N \equiv 3 \pmod 5,\ \ N \equiv 2 \pmod 7,$$

and we seek the least integer value of N. (The answer is 23.)

The chapter also has a brief discussion of mathematics in Japan, notably that of Seki Takakazu (1642–1709). Here was a mathematician who "discovered determinants ten years before Leibniz, . . ., discovered the conditions for the existence of positive and negative roots of polynomials, did innovative work on continued fractions, and discovered the Bernoulli numbers a year before Bernoulli."

George Joseph's centrepiece of the book is his account of mathematics in India, and it is laid out in Chapters 8, 9 and 10. The first of these talks of ancient India, ideas from the Vedic period, Indian numerals, and Jaina mathematics. The second is on the classical period, recording the contributions by Indian mathematicians to astronomy, algebra and trigonometry (Aryabhata I, 5th century CE; Brahmagupta, 6th century CE; Mahavira, 9th century CE; Bhaskara II, 12th century CE). The third is on what might justifiably termed the crest of the peacock, the Kerala school of mathematics, especially the results attributed to Madhava (14th century CE) and Nilakantha (15th century CE). Though this is perhaps the main section of the book, I will not discuss it in detail here since much of this mathematics was described in the review of Kim Plofker's book (*At Right Angles*, Volume 3, No. 3, November 2014, pp 82–87).

The final chapter is on mathematics from the Arab world which Joseph presents as a prelude to modern mathematics. While the development of algebra in the Islamic world and its impact on European mathematics is well known, much less is generally known of the work of Islamic mathematicians in number theory, geometry and trigonometry. This brief chapter has enough material to interest every mathematics teacher in India. The work of Ibn al-Haytham (965–1039), al-Biruni (973–1051), Omar Khayyam (1048–1126), and al-Kashi (1429) are important. Of these, Omar Khayyam is famous as a poet, but he was also a first-rate mathematician who propounded a geometric theory of cubic equations and tried (unsuccessfully) to derive the parallel postulate from other axioms. Al Kashi performed prodigious calculations, computing the value of π by circumscribing a circle by a polygon having 3×10^{28} sides!

As an appetizer, let me offer Joseph's illustration of the work of Omar Khayyam in Chapter 11. Suppose that we have a 'ratio problem' with a, b, c, d such that $\frac{b}{c} = \frac{c}{d} = \frac{d}{a}$. Then, $\left(\frac{b}{c}\right)^2 = \frac{c}{d} \cdot \frac{d}{a} = \frac{c}{a}$ and hence $c^3 = b^2a$. Letting $b = 1$, if there exist c and d such that

$$c^2 = d \text{ and } d^2 = ac,$$

then we can determine the cube root c of a.

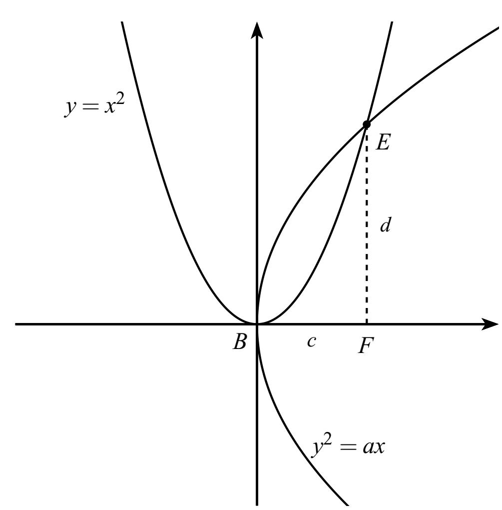

Figure 2.

Omar Khayyam perceived an underlying geometry in this problem. In the equation above, think of a as a constant and $c = x$ and $d = y$ as variables. Then we have two parabolas as in Figure 2, with equations $y = x^2$ and $y^2 = ax$; they have a common vertex B and mutually perpendicular axes, and they intersect a second time at E. It is easily checked that at E we have $x^3 = a$. Hence $BF = c$ is the cube root of a.

Khayyam extended his method to solve any third degree equation for positive roots. He solved equations for intersection of parabolas, of hyperbolas, of circle and parabola, and of parabola and hyperbola. Such application of geometric techniques to algebraic problems is of tremendous pedagogic value in the higher secondary stage in our schools, and I offer this only as a pointer to the rich lore available in mathematics from the Islamic world.

Pride and Practice

The *Peacock* is beautiful, and George Gheverghese Joseph has a pleasant style. To give you a flavour of his style, I quote from his concluding paragraph:

. . . [I]f there is a single universal object, one that transcends linguistic, national, and cultural barriers and is acceptable to all and denied by none, it is our present set of numerals. From its remote beginnings in India, its gradual spread in all directions remains the great romantic episode in the history of mathematics.

Indeed it is, and the style makes for very pleasant reading. There are some natural criticisms of the book, and since the first edition appeared in 1991, historians have pointed to several flaws: the overuse of binary opposition of 'European' vs 'non-European' mathematics, when he himself is making the case for global transmissions; speculation where there is no documentary evidence; problems with his dating; insufficient demonstration that modern mathematics was indeed as strongly influenced by these 'eastern' contributions; and so on. Clemency Montelle's review of the third edition in Notices of the American Mathematical Society, December 2013, is a good place to not only read the critique but also get pointers to more recent and authoritative historical sources.

However, it is undeniable that George Joseph is pointing us to a serious lacuna in our education, and in our teaching and learning practices. When we appreciate the cultural rootedness of mathematics and the history of mathematical thought across diverse cultures, it expands our horizons in multiple ways: rather than mere pride, we obtain a nuanced appreciation of our own past and culture, and its deep connections with other cultures; rather than accepting definitions and concepts as given (by an alien culture), we engage with them, question them and conceive of how alternate trajectories may have altered them. The vast range of examples from across the world presented by Joseph encourages us to look closer at people's practices and unearth heuristics and algorithms that, on exploration, may pose interesting questions for mathematics.

There is a convenient (albeit oversimplified) classification of mathematics encountered in learning the subject: at school we learn mathematics from the 18th century and earlier; as undergraduates, we learn largely mathematics from the 19th century; and as graduates and researchers, we approach 20th century mathematics. Even as a thumb rule, this observation yields an important lesson. If we wish to question the components that constitute school mathematics by considering alternate definitions, methods and trajectories, it's a good idea to look at the past and across cultures. *The Crest of the Peacock* offers a panoramic view of what we are sure to find.

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