Low Floor High Ceiling Tasks

Getting into Shape

Tangram Time

in the classroom

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In the November 2014 issue of *At Right Angles,* we began a new series which was a compilation of 'Low Floor High Ceiling' activities. A brief recap: such an activity comprises a sequence of tasks which are fairly easy to begin with and can be attempted by all the students in the class. However, the tasks progressively become more difficult. The objective is to challenge the problem-solving skills of students and in attempting them, each student is pushed to his or her maximum potential. There is enough work for all but as the level gets higher, fewer students are able to complete the tasks. The point, however, is that all students are engaged and all of them are able to accomplish at least a part of the whole task. In the first part of the series (in the November 2014 issue), we looked at pentominoes and in the March 2015 issue at the Fibonacci series and the regular pentagon. This time we turn to an old favourite: tangrams!

Keywords: *tangram, triangle, quadrilateral, square, parallelogram, rectangle, congruence, similarity, collaboration*

This is how Wikipedia describes this popular puzzle. The **tangram** (Chinese: 七巧板; pinyin: *qīqiǎobǎn*; literally: “seven boards of skill”) is a dissection puzzle consisting of seven flat shapes, called *tans*, which are put together to form shapes. The objective of the puzzle is to form a specific shape (given only an outline or silhouette) using all seven pieces, which may not overlap. It is reputed to have been invented in China during the Song Dynasty, and then carried over to Europe by trading ships in the early 19th century. It became very popular in Europe for a time then, and then again during World War I. It is one of the most popular dissection puzzles in the world. A Chinese psychologist has termed the tangram “the earliest psychological test in the world”, albeit one made for entertainment rather than analysis [1].

Tangram tasks are appropriate for students of class 6 and upwards. As usual, each card (or set of cards) is a task which features a series of questions which build up in complexity. Since concepts such as congruence or similarity are dealt with only from class 7 onwards, it is possible that the teacher may have to select tasks which are appropriate for the students. However, these activities also provide a platform for a gentle and informal introduction to concepts taught at a higher grade.

TASK 1: To identify and describe the 7 Tangram pieces separately

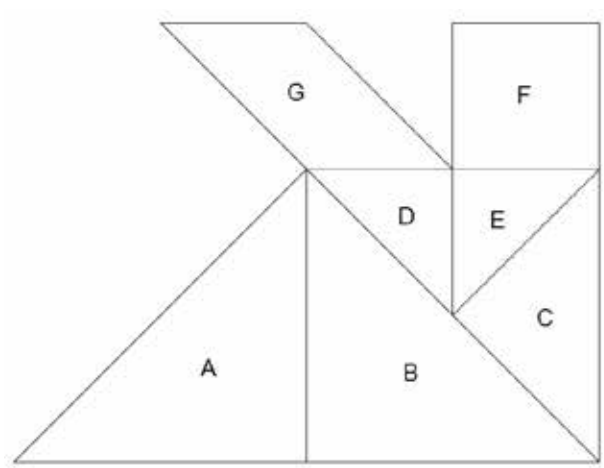


Fig. 1

Look at this tangram set (Fig. 1).

- Place tracing paper over this set and make cutouts of all the 7 shapes (A-G).
- Arrange the cutouts in sets of similar shapes.
- Label each set and describe the properties common to the members of each set.
- If there is more than one piece in a set, classify this set into subsets of congruent shapes. Describe the relationship between the subsets of the shapes.
- Write down your observations.

Teacher’s Note: This is an easy introduction to the tangram kit for students who are not familiar with it. The aim of this task is primarily mathematical communication and documentation. It also helps students review and distinguish between terms such as ‘congruent’, ‘similar’, ‘set’, ‘subset’, ‘members’, etc. Knowledge and understanding of the basic angles and shapes is a necessary prerequisite for this task. The terms ‘congruence’ and ‘similarity’ may be replaced by ‘replica’ and ‘scaled up/down version’ respectively.

TASK 2: To reassemble the Tangram kit and describe the kit as a whole

For this task, the class may be divided into two groups. Each group is given one of the pictures shown (either Fig. 2.1 or Fig. 2.2) and not allowed to look at the other picture. Instructions for both groups remain the same except where specified.

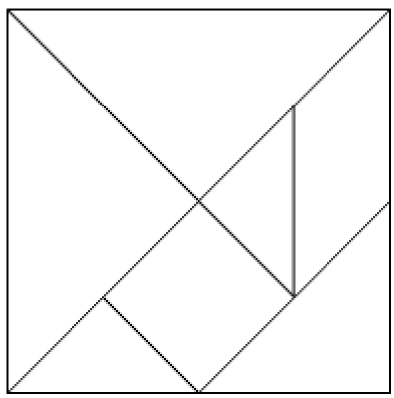


Fig. 2.1

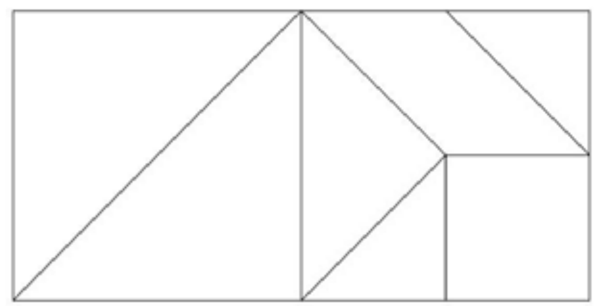


Fig. 2.2

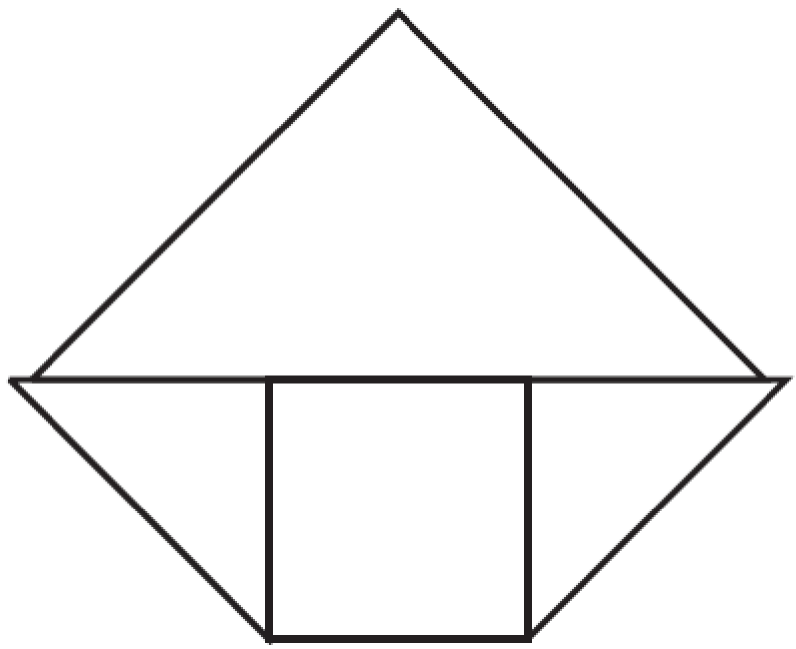
- Re-assemble your cutouts to make the picture your group was given. Identify the final assembled quadrilateral. Label the polygons A-G with the same labels given in Task 1.
- Copy your quadrilateral (Fig. 2.1 or Fig. 2.2), with all the parts A-G fitting, into your notebook. Write down a set of instructions with which this picture can be re-created by someone who hasn't seen it before.
- Group 1: Working with a partner from group 2, dictate your set of instructions to your partner and check if he/she was able to draw Fig. 2.1.
- Group 2: Now, working with the same partner from group 1, dictate your set of instructions to your partner and check if he/she was able to draw Fig. 2.2.
- You have identified the geometrical shapes A-G in Task 1 and described their properties. Discuss with your partner and write down why the instructions given produced the shapes A-G and made them fit together.

Teacher's Note: The objective of this task is for students to see how the 7 pieces of the tangram fit together to make a square or a rectangle. Once either quadrilateral is assembled, they also have the opportunity to see how it can be deconstructed into the original 7 pieces. The instructions they write should produce the very same shapes A-G. Labelling the four vertices of the square or rectangle as well as new points on the sides (beginning from H to avoid confusion) will bring better clarity to the instructions and may be suggested to the students. Here is the opportunity for peer assessment of mathematical communication skills. Feedback is instant and corrective measures can be suggested by peers to improve work.

For those with access to dynamic geometry software, such as GeoGebra, this can also work as a lab exercise to create the sketch with the instructions that have been noted down. The last question is definitely a stab at the high ceiling and may not be attempted by all, as it requires students to use properties such as: the diagonals of a square intersect at right angles. But it sets the foundation for developing the skills of proof. After the attributes of each of the 7 shapes are checked using either the compass box or GeoGebra, students can practice their reasoning, logic and knowledge of geometry by explaining why the instructions they gave produced these shapes.

TASK 4: To study the lengths of the sides of different shapes and to use Pythagoras' theorem.

- Taking the side of the square as unit length, find the lengths of the sides of the small triangle and the biggest triangle.
- Express the hypotenuse of the big triangle in 2 different ways.
- Examine and re-create the following picture with your tangram pieces:



Why is the hypotenuse of the big triangle slightly less than 3 times the sides of the square?

Teacher's Note: A necessary prerequisite for this task is the knowledge of Pythagoras' theorem and the understanding and manipulation of irrational numbers. Students should be guided to arrive at $\sqrt{8} = 2\sqrt{2}$ which is the purpose of stressing on two different ways to arrive at the hypotenuse. In the second part, students get practice in approximating the value of an irrational number, which is also an exercise in mathematical communication, reasoning and logic.

TASK 4.1: To assemble the individual pieces into geometrical shapes.

- In how many ways can you make a square i.e. with
  i. 1 piece
  ii. 2 pieces ....and so on.

Draw each configuration.

Teacher Note: To make the square, there is only 1 possible configuration in each case with 1, 3, 4, 5 and 7 pieces. With 2 pieces, 2 configurations are possible where the square can be big or small with the respective pairs of identical triangles. This task may be done as a paired activity as students will benefit from discussion and will be able to share their understanding about the properties of each shape.

TASK 4.2

- Which of the following shapes: triangle, rectangle, parallelogram, isosceles trapezium and trapezium can you make using:
  - i. Any of the pieces
  - ii. Any 2 pieces
  - iii. Any 3 pieces
  - iv. Any 4 pieces
  - v. Any 5 pieces
  - vi. Any 6 pieces
  - vii. All 7 pieces
- Draw each configuration and complete the following table

No. of pieces →	1	2	3	4	5	6	7
Triangle	✓						
Square	✓						
Rectangle	×						
Parallelogram							
Trapezium							
Isosceles trapezium							

Teacher's Note: For shapes other than the square, more than one configuration is possible with the same number of pieces. This task can also be assigned as group work as time constraints may restrict a student from coming up with all the options. Students are called upon to use the properties of each shape, most often intuitively rather than overtly. The table helps them put down their findings systematically.

TASK 5: To prove that a certain configuration is impossible

In Task 4, which shapes could you not make with 6 pieces?

Prove that it is impossible to make these shapes with any combination of 6 pieces.

Teacher's Note: Making a conjecture and then proving it is a sophisticated mathematical skill.

Conjecture: It is not possible to make a square or a right isosceles triangle with any 6 pieces of the tangram set.

Proof: Let us assume that the side of the square is of unit length. The areas of the 7 pieces are given in the following table. The total area = 1 + 2 × ½ + 1 + 2 × 2 + 1 = 8 square units. In the following table we enumerate different cases where a shape is made by leaving out one of the 7 pieces.

Piece	Sides	Area
Square	1	1
Smallest triangles	1, 1, $\sqrt{2}$	½
Medium triangle	$\sqrt{2}$, $\sqrt{2}$, 2	1
Biggest triangles	2, 2, $2\sqrt{2}$	2
Parallelogram	1, $\sqrt{2}$	1

Piece left out	Area of any shape made with the remaining 6 pieces
Biggest triangle	8 – 2 = 6
Medium triangle, square, parallelogram	8 – 1 = 7
Smallest triangle	8 – ½ = 7½

Thus, if any combination of 6 pieces are used to create a square, it will have an area of 6, 7 or 7½, hence it will have sides of $\sqrt{6}$, $\sqrt{7}$ or $\sqrt{(7½)}$ = ½$\sqrt{30}$.

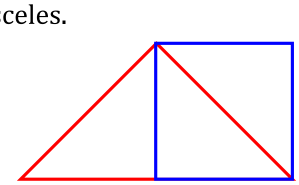
But with the pieces of the tangram set, we can only have sides of the form $a + b\sqrt{2}$ where $a, b = 0, 1, 2, 3$ … (1)

This automatically rules out squares of area 6, 7 or 7½. Since neither $\sqrt{6}$ nor $\sqrt{7}$ or $\sqrt{30}$ can be expressed in the above form (1), we conclude that it is not possible to make a square with any 6 pieces.

The case of the triangle is very similar once we prove that it has to be right isosceles.

Any triangle must have 2 acute angles.

The only possible acute angle in any of the tangram piece is 45°.



$\therefore$ any triangle made of tangram pieces must have 2 angles of 45° each and therefore the 3rd angle must be 180° – (45° + 45°) = 90°

$\therefore$ it must be a right isosceles triangle.

Its area will be ½ base × height = $\frac{1}{2}x^2$ = 6, 7 or 7½ which means that x must be $\sqrt{12}$, $\sqrt{14}$ or $\sqrt{15}$ which as we saw is not possible with the pieces of the tangram set.

So we can conclude that it is not possible to make a triangle with any 6 pieces.

Conclusion

Tangrams have always been presented as fun and hands-on activities for the class and that is what we have aimed for with this set of low floor high ceiling tasks. But we have raised the bar with an added layer of proof which is aimed at developing students' reasoning, logical and communication skills. As always, the teacher should facilitate discussions but give the students time to develop their own line of thought - they are sure to surprise and delight you!

References

1. https://en.wikipedia.org/wiki/Tangram (downloaded on September 13, 2015)

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