Azim Premji University

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TEACHING Word Problems

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A PRACTICAL APPROACH

At Right Angles
A Resource for School Mathematics

Word problems become a stumbling block for many children, including those who are adept at operational and procedural skills. Many children develop an approach to tackling word problems based on looking for cue words – such as altogether, difference, sum and so on; but this has a very limited value. Too often, such children resort to guesswork while figuring out an operation. These children experience significantly greater math anxiety when they are confronted by word problems. Why is this?

Primary reasons

Here are some reasons which lie behind such math anxiety:

1. Lack of exposure to problem situations and problem contexts during the introductory and teaching phase.
2. Lacunae in the usage of concrete materials as an aid in the visualisation of the problem.
3. Insufficient training in representation of problems through drawings and other means of reconstruction.
4. Difficulty in following multiple statements and instructions at the same time.
5. Inadequate stress on vocabulary and weak linkages or connections between concepts and associated words.
6. Absence of discussion and conversation around the questions (whether in English or in the mother tongue).
7. Lack of recording of the solution by the children in their own words. Most teachers follow rigid ways of writing statements for word problems. Writing in the initial years must come from the child's own experience and understanding. It need not be structured according to any norms; on the contrary, it needs to be personal.

All of these reasons point to poor teaching practices.

In conjunction with this is the fact that many textbooks are not particularly child-friendly. By the time the child reaches class 4 or 5, he or she would have basic literacy skills. Yet very few children read the textual material for the following reasons:

1. The language used is not close to the child's experience.
2. The word problems are not based on real life and familiar situations.
3. They are not phrased in a sufficiently interesting way, and do not draw the child 'into' the problem.
4. They are not accompanied by drawings (this is crucial for non - English-speaking learners).
5. They are often limited in variety and repetitive, and thus hold no challenge.

Often the problems are not posed in a properly graded sequence.

Keywords: *Math anxiety, thinking skills, logic, reasoning, exploring, pattern, conjecture, experiential*

Language matters

Language plays an important role in learning mathematics. Several reasons can be listed to see why:

1. In the context of mathematics teaching, a teacher uses language to communicate concepts and processes, to provide explanations, to compare alternatives, to discuss causal and dependent relationships, and to justify answers.

   In a country like ours, the language used in the classroom may not be the mother tongue of the child. Hence, particularly in the primary school, the mathematics teacher may need to translate to aid in the comprehension of the problem.

2. It is important for the child to articulate his understanding through language so that the teacher gains a window into the child's interpretation and understanding of the problem. Language and discussion enable the child to reconstruct the problem.

3. Words tend to acquire special meaning in the context of a math problem. Words like measure, share, part, product, equal, face, table and volume carry other meanings in normal, everyday language, but in a mathematics classroom may have a somewhat different connotation. Some words have more than one meaning in mathematics, e.g.: base, difference, square. Children need to become familiar with their usage in math contexts.

   Teachers need to use correct language while teaching. For some reason, in India there is a practice of using 'into' instead of 'times' while teaching multiplication table. So '2 multiplied by 3' should be read aloud as 'two times three' (and not '2 into 3' which implies division).

   Also, in problems involving subtraction, the right word to use is 'exchanging' tens for units or hundreds for tens, etc, and not 'borrowing'.

4. Mathematical symbols have to be initially understood through words, and children need to acquire clarity about them. One of the frequently misunderstood symbols is the 'equals to' symbol. It does not imply 'write the answer'. However, given the problem 2 + 5 = __ + 4, many children may fill the blank with a '7'.

5. The child has to learn to read, comprehend and interpret the text and word problems.

An important point I wish to emphasise is that an inability or weakness in attempting word problems is not an isolated problem. The teacher needs to realise that the root of the problem lies in the teaching process itself. Often when children struggle with a word problem, it serves as a pointer to the fact that either the concept or the math terminology itself has not been explained adequately. As a teacher, when I introduce a topic, I need to use varied real life contexts and situations to highlight the connections between the real world and the problems posed in the class. I need to verbalize the multiple ways of looking at a problem or analysing the structure of a problem. As I explain procedures or draw diagrams, I need to give a running commentary to show the relationship of my action to the wording of the problem. Most importantly, I need to help the child to 'get inside' the problem.

When does one start helping children to feel their way into word problems?

Apart from the exposure that happens at home, it is almost from day one in preschool and primary school. Preschool classrooms usually have equipment which lends itself to pretend-play activities (dressing up, cooking, shopping), and these can be used to introduce mathematical vocabulary. "Arya has a bigger glass than Disha", "Warad is wearing a longer kurta than Sarthak", "Mitali's shop has more flowers than Shreyan's", etc.

Number rhymes with math contexts are to be acted out so that the child sees the connection between the words he or she is reciting and the action associated with it. Rhymes which teach number concepts, ordinal numbers, increasing or decreasing sequences of numbers can be taught as action songs to demonstrate the underlying simple concepts.

Real life situations which occur during the school day of a child (lining up for assembly, library time, games time, recess time when snacks are served) should be made use of to teach the appropriate vocabulary and to introduce mathematical concepts. These everyday situations can be later referred to and incorporated into the class conversations.

Measurement-related vocabulary (big, bigger, biggest), the vocabulary of comparison (more, less), the vocabulary of quantity (numbers) and other such words are taught through activities and actions.

The notion of time is complex to grasp, and the related vocabulary (yesterday, today, tomorrow) is best learnt when children begin to share their experiences in the class and the teacher reinforces the concepts by talking about the planned activities for the day and the following day, etc.

It is very important to create enjoyable action settings, where children feel secure and free to ask questions. This provides children the opportunity to communicate, to comment on the activity they are performing, and to formulate questions. The teacher must step in to supply them with the right mathematical vocabulary as they struggle to express themselves.

GAME SETTINGS

Games provoke talk, reactions and arguments. They offer excellent platforms for reinforcing concepts and associated vocabulary.

Bogie game: Write some random numbers on the floor tiles.

Figure 1

Children can move from one bogie to the next by making a statement using appropriate mathematical words. The child standing on the first tile says 'I need seven more to go to the next bogie', the child standing on the second tile says 'I am 3 more than the next bogie', the child standing on the third tile says 'I will give away 4 to go the next bogie' and so on. As the focus of the game is on using language, a child who makes a mistake in his or her calculation can be corrected and allowed to move on.

I share here some approaches I have used in helping children to attempt and solve word problems. In all the approaches, one essential aspect is the slowing down of the reading of the word problem. Children find it difficult to hold multiple statements all at once in the mind. These approaches help in breaking it down for them. They learn how to codify the facts and interpret each statement separately. Through the process of reconstruction, they begin to understand the problem as a whole and the way the parts relate to each other.

I have illustrated the various approaches through examples selected from different levels of lower primary school.

One
Dramatization and mime

Most children enjoy drama and role playing. It doesn't take very much time or resources to act out a word problem set in a context. Blocks (interlocking cubes are more useful as they stack up well and can be easily used for comparisons by stacking) can be used to represent small numbers, and can easily be scattered or collected in an action scene.

Note: Larger numbers can also be represented by using a bigger block to stand for ten.

LEVEL: CLASS 3, 4

The children or the teacher can read out the problem to the class. For young children, the teacher may need to read out the problem or guide them in their reading.

Meher has three guavas. Aryaman has four more guavas than Meher. Their friend Adway brings two guavas. They share the guavas equally amongst themselves. How many guavas does each one get?

The problem is about guavas which are a great favourite with children. They love to climb guava trees and pluck guavas. The teacher needs to initiate a discussion to stimulate interest in the problem before expecting children to enact out a story. The discussion can begin with a question: 'What is your favourite fruit?', 'Shall we bring some guavas from a fruit garden?' and so on. It is important to draw the children into the context of the problem for it to become real.

Let three children act out the whole scene. They pretend to be in a garden, plucking guavas from the trees. Initially Meher shows her three guavas with blocks and states 'I have three guavas'. Aryaman has to show seven blocks and state 'I have four more guavas than Meher'. The remaining children can check whether he has got the right number. If the problem has not been read carefully, or if the child has not understood the statement correctly, he may show only four blocks. At this point, either the other students may point out the error, or the teacher can intervene. Now Adway can pretend to bring in two guavas. Together, they now need to figure out how to share them equally. They can finally state 'We have twelve guavas altogether. Each of us gets four guavas. Three fours make twelve.' The focus of this activity is the following: (i) To interpret the statements correctly; (ii) to make appropriate statements corresponding to their actions.

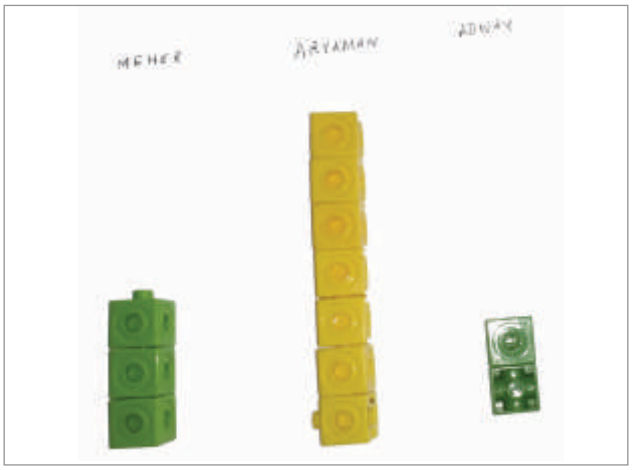


Figure 2

Modification of the question: Having spent time drawing the children into the garden theme, the teacher can play around with the question by changing the numbers or the operations involved in the following way.

'What if Aryaman finds worms inside two of them and throws them away? How many more does Aryaman now have than Meher?' (Both can stack up their fruits to find the answer.)

'Will they now have enough guavas to share equally?'

'If they need to take 15 guavas back to their class, how many more guavas do they need to pluck?'

What does dramatization of a problem achieve?

- It aids in the comprehension of the problem.
- It helps children whose reading levels are low.
- It creates a sense of participation among the children.
- It can also help a teacher to assess a student's comprehension quickly
- Teacher can initiate remedial measures quickly by simplifying a problem or raising the challenge level.

Two
Using concrete materials

Solving problems with the use of concrete materials shares some aspects of dramatization as well as mathematical modelling. However, it is particularly relevant for word problems which require special equipment for modelling; for example, place value materials, cardboard clock, play currency or geometric shapes.

LEVEL: CLASS 3

Vaishnavi and Ameya had 16 straws to make triangle shapes. They used three straws for each triangle. How many more straws do they need if they want to make 6 triangles altogether?

Children can use sticks to model the problem. It happens on occasion that as children begin the process of modelling, they begin to visualise the problem and are able to work out the solution even before completing the act of modelling. In such cases, it is important to let the children state the answer at that point itself, and get them to verify the answer by completing the act of modelling.

Figure 3

LEVEL: CLASS 4

Samriddhi made a string of beads, one red bead followed by two yellow beads. What will be the colour of the thirteenth bead?

A child may use coloured counters to model such a problem and find the answer.

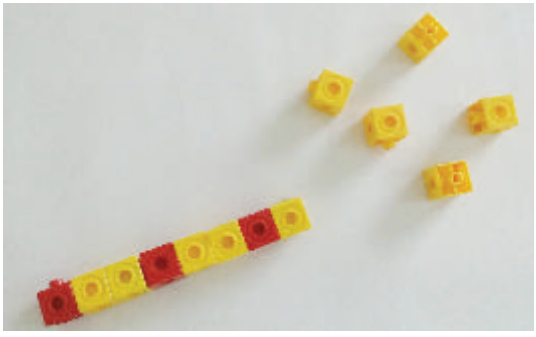


Figure 4

As the children play around with the models, the teacher can suggest trying out new number combinations. What if you make a string with two red beads followed by three yellow beads? What will be the colour of the tenth bead? They can model this situation and record the result in a sentence form.

My string has two red beads followed by three yellow beads. The thirteenth bead is _____.

Children can try out their own combinations and record the result in sentence form.

What does usage of concrete material achieve?

- It facilitates tactile learning and development of kinesthetic intelligence.
- It simplifies the problem and aids in the comprehension of the problem.
- It serves as a visual aid in solving the problem.
- Associated actions like combining things or comparing things will help the child to figure out the operation involved.
- The focus is not on the solution itself, but on modelling and comprehending the problem.

Three
Using drawings

Some word problems are better understood by representing them through drawings.

LEVEL: CLASS 3

Diva was making a border pattern with squares and circles. She drew 10 squares and made 2 circles between every 2 squares. How many circles did she make?

LEVEL: CLASS 5

It takes 10 minutes to saw a log of wood into two pieces. How long does it take to saw the same log into four pieces?

In some of these problems, the children will need to make suitable pictures to understand the problem. They need to perceive the patterns and relationships before they can find the answer.

Making pictorial representations helps in comprehending a problem. However, children may benefit from help in learning how to use pictures as an aid to solving a problem. A teacher can show different ways of representing word problems through drawings. The choice of type of drawing will be based on the context of the problem. Here are a few models.

Pictorial drawings:

LEVEL: CLASS 2

Jhanvi has black and grey marbles. There are 10 marbles in all. If 6 of them are black marbles, how many are grey?

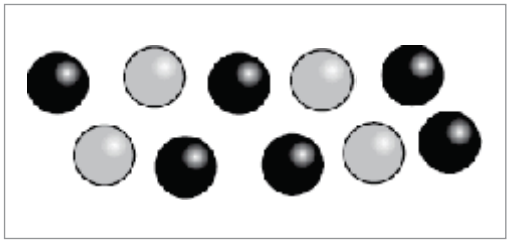


Figure 5

The blue box is bigger than the yellow box. The blue box is smaller than the black box. The black box is smaller than the red box. Which is the biggest box and which is the smallest box?

Figure 6

Children can be encouraged to make symbolic pictures for these problems before trying to solving them.

Since children typically will not be able to read at this age, in the follow-up sessions the teacher can supply the children with well-illustrated worksheets.

LEVEL: CLASS 4

In the school dining hall, there are 8 tables with 12 people on each table, 6 tables with 4 people on each table, and 2 tables with 9 people on each table. How many tables are there? How many people are there altogether?

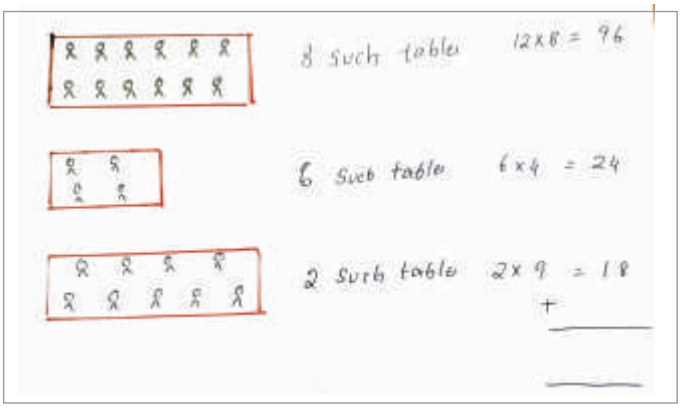


Figure 7

Number line drawings: Horizontal and vertical lines can be used in many kinds of problem settings.

LEVEL: CLASS 4

Rahul is standing behind Yagya in the queue. Rahul is the fourth child in the queue from the front. Yagya is the seventh child from the back of the queue. How many children are there in the queue?

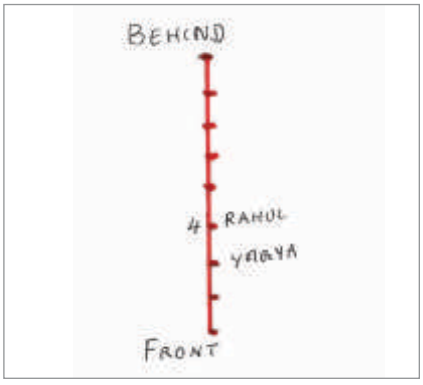


Figure 8

LEVEL: CLASS 4

Aum is four years old. His sister Shreya is seven years old. After five years, how much older will Shreya be than Aum?

This is a question based on common sense. But many children resort to totalling the numbers and they come up with incorrect, meaningless answers as they do not comprehend the problem properly. A number line will make the problem less abstract, and help them realise that the age gap remains the same.

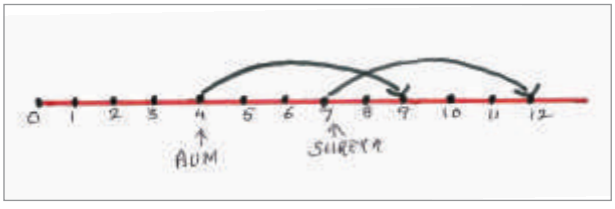


Figure 9

LEVEL: CLASS 4

Sujoy's house is on the third floor of an apartment block. Between every two floors there is a flight of 15 steps. How many steps must Sujoy climb to go from the ground floor to his home?

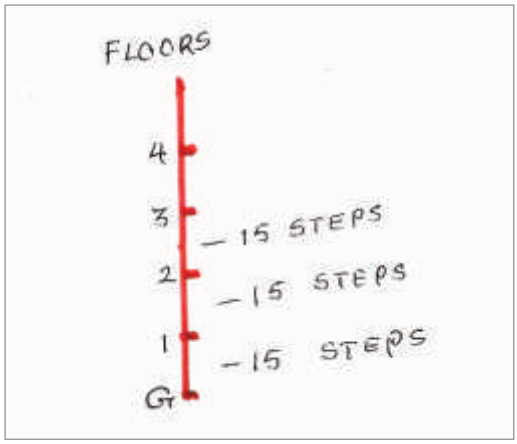


Figure 10

LEVEL: CLASS 5

A bookshelf has five levels. The third level has ten more books than the second level. The second level has four fewer books than the first. The fourth and fifth levels together have the same number of books as the third level. If the first level has 18 books, how many books are there in all in the bookshelf?

Flowchart drawings:

Some problems lend themselves well to flowchart drawing as shown in Figure 11.

LEVEL: CLASS 5

I think of a number. I multiply it by 2, divide by 3, and subtract 4. The result is 2. What is my number?

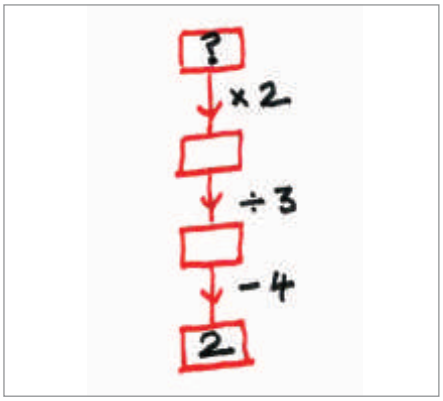


Figure 11

LEVEL: CLASS 5

Chinmayee has four 1 rupee coins, two 2 rupee coins and a 5 rupee coin. In how many ways can she pay for a fruit that costs Rs. 9?

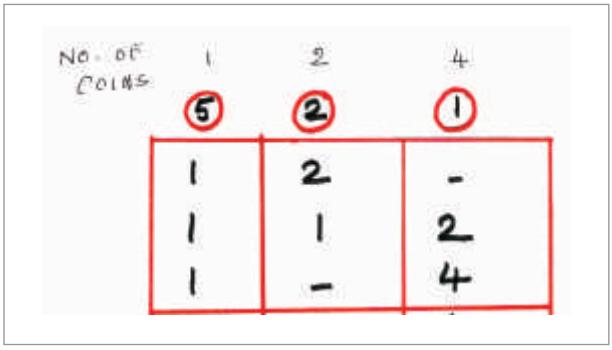


Figure 12

Branching drawings:

Samarth and his friends are making a pyramid formation for their school Sports Day. Samarth balances himself by standing on two students. Each of those two students stands on two other students, and each of those students stands on two other students. How many students are there in all in the whole formation?

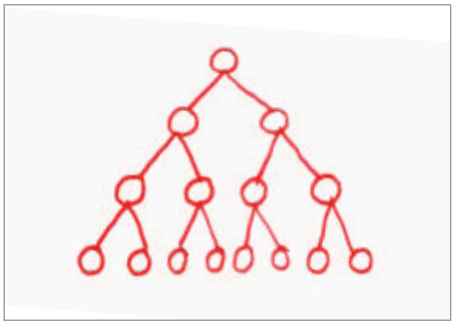


Figure 13

Bar drawings:

A bar drawing is a very useful tool for problem solving which can be used at various levels, right up to high school. It is highly versatile as it can be adapted to various types of problems. Students should be introduced to this method in a graded manner as detailed here. They should practice with several problems at each stage to make the transition smoother.

Stage 1: Use linear pictures to show an operation.

Ex. Addition

Ashwin has 4 cupcakes, and Akriti has 6 cupcakes. How many cupcakes do they have altogether?

Figure 14

Tabular drawings:

Some problems are better comprehended by using a tabular approach. All possible combinations can be drawn using a table format as shown in Figure 15.

Stage 2: Draw bars around the pictures. The part-whole relationship is brought into focus. The parts have 4 cupcakes and 6 cupcakes. Together they make the whole which is 10 cupcakes.

Figure 15

Stage 3: Replace pictures with dots. Dots are a symbolic representation for pictures. Draw arrows below the bars to focus on the whole.

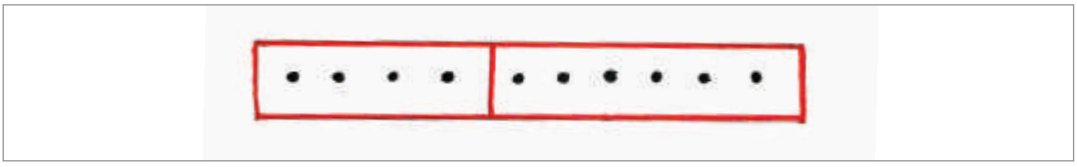


Figure 16

This is to help make the transition from the semi-concrete (picture) to number form.

Stage 4: Replace dots with numbers for the parts. The arrows below the bars represent the whole.

At this point take care to draw bigger bars for bars whose value is high. This will aid in the visualisation.

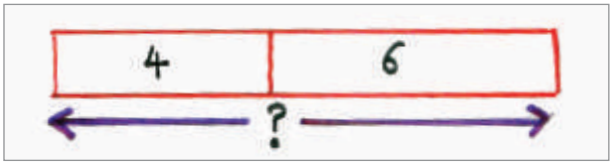


Figure 17

Stage 5: Draw multiple bars to show comparisons. The arrows can also be used to point to portions of the bars. Totalling or summing can also be indicated clearly as shown in these figures.

Ex. Comparison

Kriti has 8 toffees. Nitya has 7 more toffees than Kriti. How many toffees does Nitya have?

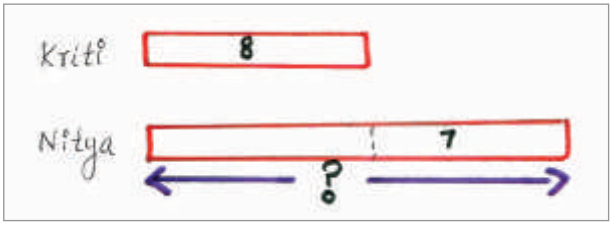


Figure 18

Kriti has 8 toffees. Nitya has 7 more toffees than Kriti. How many toffees do they have together?

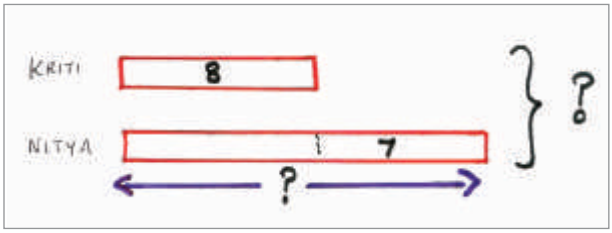


Figure 19

Tanya and Riddhi have 24 coloured pencils altogether. Riddhi has 6 pencils. How many more pencils does Tanya have than Riddhi?

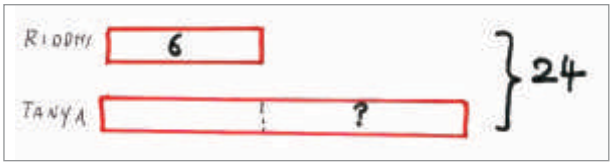


Figure 20

Ex. Multiplication

Sahil has 16 shells. Aman has 3 times as many shells as Sahil. How many shells does Aman have?

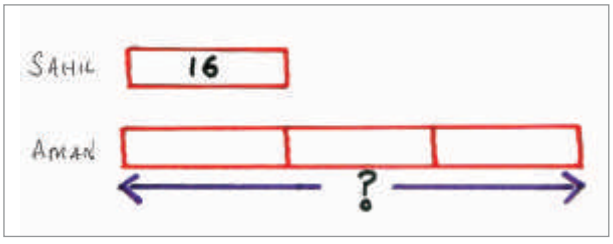


Figure 21

Ex. Division

Ayaan has 18 biscuits. He places 3 biscuits in each plate. How many plates did he fill?

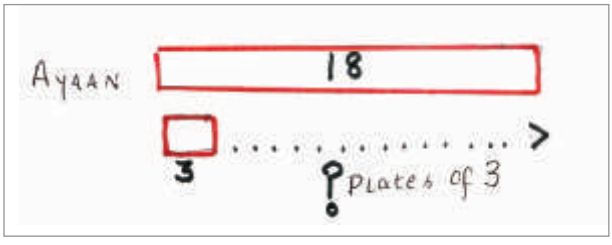


Figure 22

Ex. Fraction

One quarter (1/4) of a number is 8. What is the number?

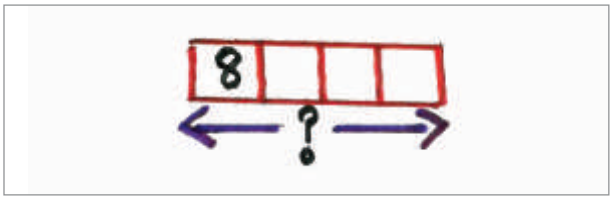


Figure 23

Stage 6: Many problems refer to a whole and its parts. An appropriate drawing can bring out the relationship between the parts and the whole clearly. For complex problems which require further partitioning of bars, it may be better to write the numbers outside the bars.

Ex. Multiple operation

Pranathi made 24 cupcakes. She ate 4 of them and gave 12 cupcakes to her friends. How many cupcakes does she have now?

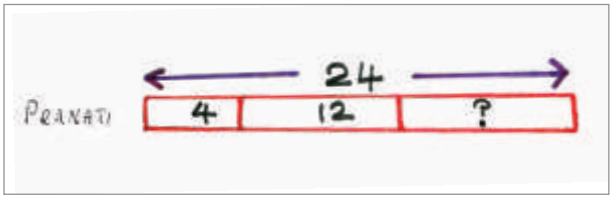


Figure 24

Two thirds (2/3) of a number is 12. What is the number?

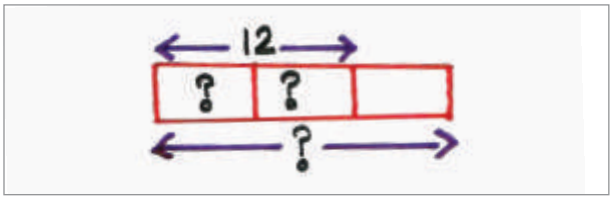


Figure 25

Class 5 has 80 students who are distributed in three sections. There are 22 students in Section A. Section C has 6 more students than Section A. How many students are in Section B?

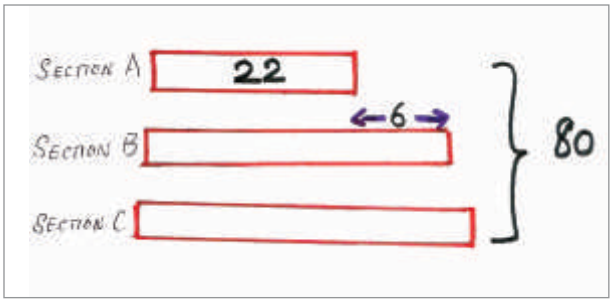


Figure 26

There are 8 more oranges than apples in a basket. There are 24 fruits in all in the basket. How many apples are there in the basket?

Figure 27

Akriti had 32 marbles. She gave half of the marbles to her brother. After that, she gave half of the remaining marbles to her best friend. How many marbles does she have now?

Figure 28

Dhruv's age is three times Aditya's age, and Hari is twice as old as Aditya. The sum of their ages is 30. How old is each boy?

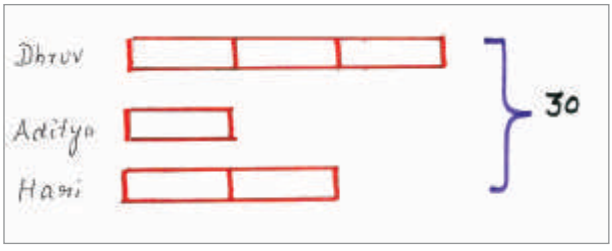


Figure 29

This problem is in fact a precursor to the algebraic way of thinking. However by using this approach, a child will be able to solve it well before he arrives at abstract thinking.

Mrs. Kapoor bought four large mangoes at Rs. 6 each, and five small mangoes at Rs. 4 each. She gave the fruit seller a fifty-rupee note. How much change does she receive from the fruit seller?

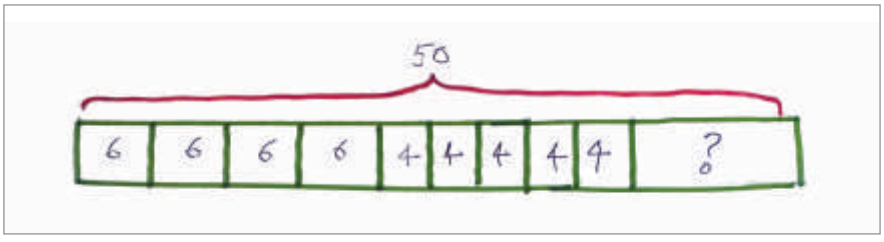


Figure 30

Four
Thinking aloud approach

Some problems lend themselves well to thinking aloud. If the teacher can solve the problem by thinking aloud slowly, children will also pick up better problem-solving techniques as well as the ability to articulate and share their thinking process. Even though young children may have difficulty in expressing themselves, with a few carefully thought-out lead questions from the teacher, they will start to verbalize their thinking. Listening to their peers often helps children notice multiple ways of looking at a problem. For the teacher, it reveals the child's understanding of the concepts and processes, or the child's misconceptions.

Level: Class 2

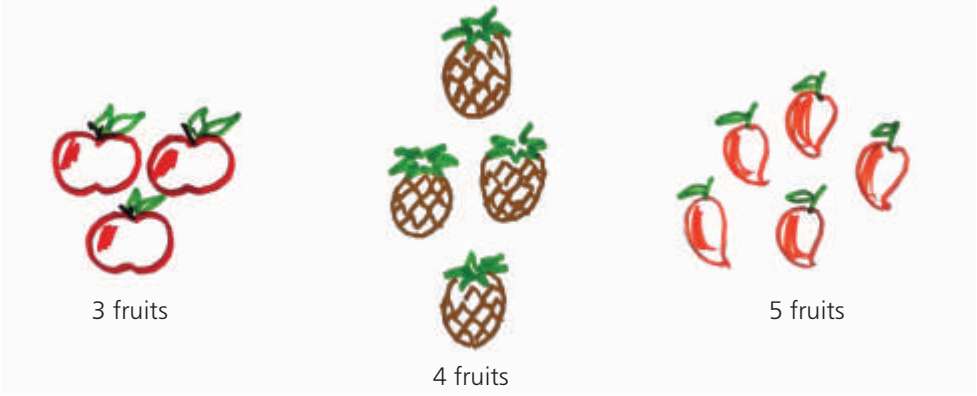


Figure 31

Here are three groups of fruits. Which two groups together have 8 fruits?

Teacher thinks aloud.

Will the first and second group add up to 8? No; 3 and 4 make 7.

Will the second and third group add up to 8? No; 4 and 5 make 9.

What about the first and third? Yes; 3 and 5 do add up to 8.

Level: Class 3

Figure 32 - 6 fruits in the first basket, 10 fruits in the second basket

How many fruits from the first basket should be moved to the second basket so that the number of fruits in the second basket is three times the number in the first basket?

What will happen if I move 1 fruit from the first basket to the second?

There will be 5 in the first basket and 11 in the second basket. 11 is not three times 5.

What if I move 2 fruits from the first basket to the second basket?

There will be 4 in the first and 12 in the second. 12 is three times 4.

A farmer has more than 14 but less than 20 eggs. If he counts the eggs in twos, there is one egg left over. If he counts the eggs in threes, there are two eggs left over. How many eggs does the farmer have?

Can the farmer have 15 eggs? If he counts in twos, he will be left with an extra egg. But if he counts in threes, there will be no eggs left over. So it cannot be 15.

Can it be 16 eggs? And so on.

Five
Writing word (story) problems for pictures and for expressions with number operations

The teacher can give a picture which depicts a problem situation. The children can make up a story problem to match the picture. If the children are not yet in a position to write, they can narrate the story orally and the teacher can write it out for them on the board. When children create and pose a problem for a picture, they come up with different stories. They will have multiple ways of looking at it. This approach potentially has great value. It places the child in the position of a problem poser.

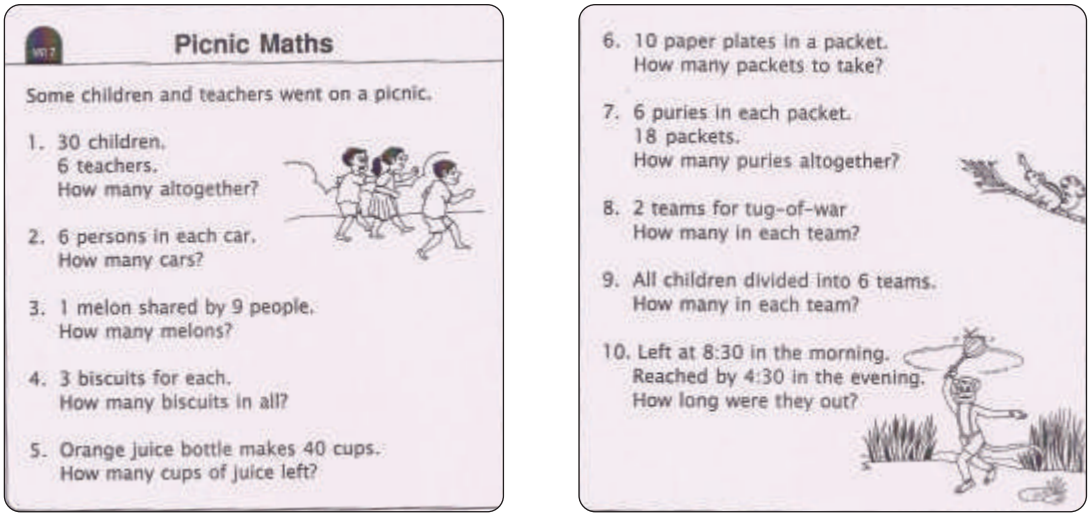
The teacher can also give simple expressions involving two or more operations and ask children to create story problems for them. For example:

Write a story problem for $15 - (5 + 7)$ and another story problem for $15 - 5 + 7$. Discuss both the stories in the class.

Figure 33

In what way do they differ?

It is also good to create theme-based word problems which are child friendly and incorporate various operations.



Picnic Maths

Some children and teachers went on a picnic.

1. 30 children.
   6 teachers.
   How many altogether?
2. 6 persons in each car.
   How many cars?
3. 1 melon shared by 9 people.
   How many melons?
4. 3 biscuits for each.
   How many biscuits in all?
5. Orange juice bottle makes 40 cups.
   How many cups of juice left?
6. 10 paper plates in a packet.
   How many packets to take?
7. 6 puries in each packet.
   18 packets.
   How many puries altogether?
8. 2 teams for tug-of-war
   How many in each team?
9. All children divided into 6 teams.
   How many in each team?
10. Left at 8:30 in the morning.
    Reached by 4:30 in the evening.
    How long were they out?

Figure 34

Six
Oral word problem approach

Oral word problem approach

On a daily basis, it is good to pose simple one-line word problems to give practice in math vocabulary and word problem comprehension. Here are a few such one liners.

1. Give me two numbers whose sum is 10.
2. How can you get a product of 12 using two dice?
3. Think of a number whose multiple is 18.
4. What is the smallest two-digit factor of 24?
5. Name some numbers which can be divided by 7 without remainders.
6. I am an odd number between 10 and 20. I am a multiple of 3. Who am I?

Write a single- or double-digit number on the board. "Tell me anything that you can think of about this number using math words." Children can use mathematical terms they have learnt: odd, even, factor, multiple, prime, composite, square number, one less than a square number, etc. Encourage them to find as many ways as describing it.

Game: Twenty questions. The teacher thinks of a number between 1 and 100. Children are allowed to pose questions (which will be answered only Yes or No), making use of math vocabulary. They should try to figure out the answer within twenty questions.

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at padmapriya.shirali@gmail.com