Rascal Triangle Synopsis

In the March 2016 issue of At Right Angles, we carried an article describing the discovery of the whimsically named Rascal Triangle. You can find the article at http://teachersofindia.org/en/ebook/rascal-triangle. We give below a brief synopsis of this story.

The story unfolds in a classroom in America where a teacher displayed the first four lines of the well-known Pascal triangle (namely, rows 0, 1, 2 and 3), and asked them to guess or to deduce what could be the next few lines.

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	?		?		?		?		?	
?		?		?		?		?		?

Table 1 : The familiar Pascal aray

What he had displayed was the array shown in Table 1

The teacher's intention was for them to discover that, in the Pascal triangle, each new row is generated additively, using the numbers in the row above it (namely, by adding the two numbers closest to the entry to be filled). Thus if we have:

a		b
	x	

then the new entry x is given by: $x = a+b$.

Instead, the students surprised him by proposing that the row after 1,3,3,1 should be 1,4,5,4,1; and the one after that should be 1,5,7,7,5,1.

In the rule used by the students, the numbers in each new row are computed using the two rows preceding it. Thus if we have:

	a	
b		c
	x	

then the new entry x is given by: $x = \dfrac{bc + 1}{a}$

If the generating rule for the Pascal array could be called a "triangular rule" (based on the underlying shape), then the one used by the students could be called a "diamond rule." The students who put forward this new rule and explored this new array whimsically named it the Rascal triangle.

1									
1	1								
1	2	1							
1	3	3	1						
1	4	5	4	1					
1	5	7	7	5	1				
1	6	9	10	9	6	1			
1	7	11	13	13	11	7	1		
1	8	13	16	17	16	13	8	1	
1	9	15	19	21	21	19	15	9	1

Table 2 : The first ten rows of the Rascal array

The article discussed the validity of this rule and proves the following:

1. Despite the division, all the entries do turn out to be positive integers.

2. Entry(k) in Row(n) of the Rascal array is $k(n-k)+1 = kn-k^2+1$.

References

1. Stover, Christopher. "Rascal Triangle."
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http://mathworld.wolfram.com/RascalTriangle.html

2. Anggoro, A., Liu, E. & Tulloch, A. "The Rascal Triangle."
http://www.maa.org/sites/default/files/pdf/pubs/cmj393-395.pdf

3. "The Rascal Triangle." http://www.maa.org/publications/periodicals/college-mathematics-journal/rascal-triangle