Middle School Mathematics

Some Reflections

ClassRoom

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Middle school maths is a convenient phrase used in textbook titles and faculty meetings and workshops for teachers. How do we distinguish the above from primary school maths, high school maths or higher secondary school maths? Primary level maths deals mostly with computation, whether it is with whole numbers or with fractions-and-decimals. Of course, a certain level of conceptualization takes place, but the emphasis is on drill and practice work so that the student will be able to calculate quickly and not get bogged down or daunted by numbers. One is trained to do the four basic operations with whole numbers as well as fractions and decimals. High school maths involves a good degree of abstraction, a lot of the content appearing to be far removed from daily life. Middle school maths in contrast is fairly grounded in the direct experience of students. There is less repetitive practice. It involves seeing patterns which can simplify situations and provide quicker pathways to solutions.

The teacher needs to be sensitive to the general temperament of students in this age group. Middle school students tend to be gregarious, enjoying social interaction, with peers as well as teachers.

The teacher should have a judicious mix of whole class discussions, group work, presentations, displays, home assignments, quiet individual work, etc., to transact the curriculum.

Keywords: *Middle school math, cooperative learning, activities, discussion, tables, LCM, GCD*

One of the key ideas/skills developed in middle school maths is the decomposition of numbers into factors, prime or otherwise – emphasizing multiplicative connections rather than additive ones. A simple way to start this is to revise the multiplication tables 'backwards'. Given a number, say 48, the student is asked to recollect multiplication facts featuring 48 as the product, in an oral exercise. The whole class could be involved in producing a factor table, where you go down the sequence of numbers, up to 100, 120 or 150, expressing each number as a product of two numbers in as many ways as possible. The students also get familiar with which numbers are prime, which numbers are composite, and which numbers are highly composite (i.e., having lots of factors).

Pedagogy: *The activity ensures drill and practice, but the approach is fresh and offers scope for discussion, is non-threatening (you may remember some tables but not all).*

This could be followed by an exercise in which one asks students to write down all the factors of a given number, including 1 and the given number, in ascending order. How does one ensure that all the factors have been taken? One could go stepwise, thus: Taking 60 to be the number in question, it is seen that 2 is a factor as also 30, 3 is a factor as also 20, 4 as also 15, 5 as also 12, and 6 as also 10. We seem to be moving to a central point from both sides. Now, since there are no factors of 60 between 6 and 10, we stop the search. This exercise leads to the observation (among others) that square numbers have an odd number of factors, while non-square numbers have an even number of factors.

Pedagogy: *It is good to emphasise the importance of individual written practice after discussion. Recording of learning helps both the student and the teacher; it helps the student reconstruct the concept or the subject matter, thus improving its registration in the mind; and it helps the teacher gauge if the student has followed the discussion.*

Pedagogy: *Building mathematical rigour and ensuring that all cases have been considered.*

Pedagogy: *Allow students to arrive at this observation – either overtly or in their practice.*

Another exercise that reinforces familiarity with numbers and their factors is based on the following observation. The product of two numbers x and y is the same as the product of the numbers ax and y/a. For instance, we could double one number and halve the other, and the product remains unchanged. Students could be encouraged to use this approach in multiplication. For instance, 48×75 may be replaced by 24×150 and then by 12×300, etc., as desired, to arrive at the product.

Pedagogy: *Strategy: laying the ground for algebraic thinking by pointing out the generalisation.*

These discussions and activities lead to a key topic at this level, LCM and GCD. Before we go for standard algorithms to obtain the LCM or GCD of two numbers, we could look at certain special cases. If the two numbers are mutually prime (this phrase needs to be defined now), then their LCM is simply their product and their GCD is 1. If one number is a factor of the other, then the greater number is the LCM and the smaller the GCD. In the general case, students can be encouraged to try to express the two given numbers as products of two numbers each, with one common factor which is as large as possible. For instance, given 36 and 48, we could express them as $36=12\times3$ and $48=12\times4$. Noting that 3 and 4 are mutually prime, 12 is the GCD and $12\times3\times4=84$ is the LCM.

Pedagogy: *Strategy of classification; understanding the difference between cases and the corresponding change in strategy.*

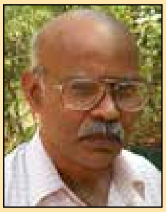
Middle school students also like to explore alternative pathways to the solution to a problem. They like to tackle multiple strategies, different groups or different individuals working on different lines and comparing results. For instance, the following problem is enjoyed by students who are often eager to try varied approaches. The task is to find the smallest number divisible by 1, 2, 3, …, 10. Some students interpret it straightaway as an exercise in finding LCM. They may take up the standard approach of expressing the numbers as products of primes. Others may go stepwise, thus: LCM of 2 and 3 is 6, LCM of 6 and 4 is 12, LCM of 12 and 5 is 60, and so on. Some others may try to cut out some numbers and simplify the situation thus: Since we have 6 in the list we can ignore 2 and 3; since we have 8 in the list we can ignore 4, and so on.

Pedagogy: *The teacher should give opportunities for students to justify their strategies thus building reasoning and logical thinking.*

An exercise with numbers (we confine ourselves to positive whole numbers for the present), which has applications in algebra is to find two numbers given their sum and product. After a few trial and error efforts one starts to choose between two approaches: splitting the 'sum' into two parts and finding their product, or expressing the 'product' as a product of two factors and finding their sum. One also notices a pattern. When we split the 'sum' into two parts and find their product, the product increases as the two parts get closer to each other. This ties up with the geometrical task of finding the rectangle with the greatest area for a given perimeter. This task could also be rounded off by preparing a graph of the observations, one of the linear dimensions (length or breadth) shown on the X-axis and the area shown on the Y-axis, to get the familiar parabolic shape.

Pedagogy: *Building HCK (horizon content knowledge), laying the foundation for concepts such as quadratics which will be taught in senior classes, also breaking the barriers between arithmetic, algebra and geometry, scope for visualisation and use of technology.*

If we think of prime numbers as atoms, then composite numbers are molecules. Getting to feel familiar with numbers and their factors and multiples is learning the composition or chemistry of numbers.



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