ClassRoom

A New Test for Divisibility by 8

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The standard test for deciding whether a given number is divisible by 8 is the following:

> *Let N denote the given number. Examine only the last three digits of N. Regard these as making up a three-digit number; call this number M. Then:*
>
> - *If M is divisible by* 8*, then N is divisible by* 8.
> - *If M is not divisible by* 8*, then N is not divisible by* 8.
>
> *In short: N is divisible by* 8 *if and only if M is divisible by* 8.

This test allows us to decide on divisibility by 8 by considering only whether a particular three-digit number is divisible by 8. Since the original number may be arbitrarily large, the test simplifies our task by reducing our labour.

Could there be a test for divisibility by 8 which involves even less work than the above test? The surprising answer to this is: **Yes**. As per the report which appeared in a national newspaper ([1]), such a test has been devised by mathematics teacher **Sursinh Parmar**, of Kodinar Taluq, Saurashtra. On September 6, 2016, Parmar received the Best Teacher Award for this innovation. Reportedly, the discovery came about as a result of a query put by a child in class 5, who asked in effect whether the standard

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test for divisibility by 8 could be carried out with fewer divisions. In an interview to the newspaper, Parmar stated: “The query baffled me and forced me to think …. I had no answer to the query ….” After wrestling with the problem for a while, he came up with the test described below.

Notation for divisibility. First we describe a convenient short-form notation that we shall use throughout this article: $a \mid b$ means that a is a divisor of b; i.e., b is divisible by a. (Examples: $4 \mid 12$; $5 \mid 35$.) The notation $a \nmid b$ means that a is *not* a divisor of b; i.e., b is not divisible by a. (Examples: $2 \nmid 5$; $3 \nmid 10$.)

Algorithm. Let the given number be N. As in the standard test, form the number M made by using its last three digits. Write M as abc where a is the Hundreds digit, b is the Tens digit, and c is the Units digit of M.

Step 1: Check whether or not $4 \mid bc$. If not, conclude that $8 \nmid N$.

Step 2: If bc is divisible by 4, compute the quotient $q = bc \div 4$.

Step 3: If a and q are both odd or both even, conclude that $8 \mid N$; else, that $8 \nmid N$.

Examples.

Ex 1: Let $N = 1,003,496$; then $M = 496$, $a = 4$, $bc = 96$, $bc \div 4 = 96 \div 4 = 24$. Since 24 and 4 are both even, we conclude that $8 \mid N$.

Ex 2: Let $N = 2,842,536$; then $M = 536$, $a = 5$, $bc = 36$, $bc \div 4 = 36 \div 4 = 9$. Since 5 and 9 are both odd, we conclude that $8 \mid N$.

Ex 3: Let $N = 6,042,586$; then $M = 586$, $a = 5$, $bc = 86$. We observe that $4 \nmid bc$; therefore $8 \nmid N$. (In this example, we did not even get past Step 1.)

Ex 4: Let $N = 6,042,588$; then $M = 588$, $a = 5$, $bc = 88$, $bc \div 4 = 88 \div 4 = 22$. Since 5 is odd whereas 22 is even, we conclude that $8 \nmid N$.

The proof of correctness of the algorithm is given at the end of the article. Further streamlining may be attempted by studying patterns in the multiples of 4.

Extension: Test for divisibility by 4. It is instructive to connect the above algorithm with the well-known test for divisibility by 4:

> *Let N denote the given number. Examine only the last two digits of N. Regard these as making up a two-digit number M. Then: (i) if* $4 \mid M$*, then* $4 \mid N$*; (ii) if* $4 \nmid M$*, then* $4 \nmid N$*. In short:* $4 \mid N$ *if and only if* $4 \mid M$*.*

We can streamline the algorithm in the light of Parmar’s idea as follows. Let the given number be N. Form the number M made by using its last two digits. Write M as bc where b is the Tens digit, and c is the Units digit of M.

Step 1: Check whether $2 \mid c$. If not, conclude that $4 \nmid N$.

Step 2: If $2 \mid c$, compute the quotient $q = c \div 2$.

Step 3: If b and q are both odd or both even, conclude that $4 \mid N$; else that $4 \nmid N$.

This may be stated much more compactly as follows:

> An even number is divisible by 4 if and only if the Tens digit and half the Units digit are either both odd or both even.

Extension: Test for divisibility by 16. In the other direction, we may also generate, along much the same lines, a streamlined test for divisibility by 16.

Let the given number be N. Form the number M made by using the last four digits of N. Write M as $abcd$ where a is the Thousands digit, b is the Hundreds digit, c is the Tens digit, and d is the Units digit of M.

Step 1: Check whether or not $8 \mid bcd$. If not, conclude that $16 \nmid N$.

Step 2: If $8 \mid bcd$, compute the quotient $q = bcd \div 8$.

Step 3: If a and q are both odd or both even, conclude that $16 \mid N$; else, that $16 \nmid N$.

Proof of correctness of the algorithm for testing divisibility by 8. The proof hinges on the following simple observations which may be easily verified:

- $4 \mid 100$; $8 \nmid 100$; $8 \mid 200$. So 8 is a divisor of all even multiples of 100, but a non-divisor of all odd multiples of 100. Further, since 100 leaves remainder 4 on division by 8, it follows that every odd multiple of 100 leaves remainder 4 on division by 8.
- Let bc be a two-digit multiple of 4, and let q be the quotient in the division $bc \div 4$. If q is even, then it must be that bc is a multiple of 8; and if q is odd, then it must be that bc leaves remainder 4 on division by 8.

The proof of the test for divisibility by 8 follows from these observations. Let $M = abc$ be a three-digit multiple of 4, and let q be the quotient in the division $bc \div 4$. Note that $M = a00 + bc$. These two numbers ($a00$ and bc) are either both multiples of 8, or both leave remainder 4 on division by 8. If M is to be a multiple of 8, then one of the following must happen:

- Both $a00$ and bc are multiples of 8. This will be the case if a and q are both even.
- Both $a00$ and bc leave remainder 4 on division by 8. This will be the case if a and q are both odd.

The logic behind the algorithm should now be clear. □

References

1. Innovative maths teacher from Saurashtra to receive award, http://timesofindia.indiatimes.com/city/rajkot/Innovative-maths-teacher-from-Saurashtra-to-receive-award/articleshow/54016478.cms, Accessed 10 September, 2016

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