TearOut

Fun with Dot Sheets

Beginning with this issue, we start the TearOut series. In this article, we focus on investigations with dot sheets. Pages 1 and 2 are a worksheet for students, pages 3 and 4 give guidelines for the facilitator

ClassRoom

Remember: Whenever a line has to be drawn, two grid points must be identified through which this line passes.

1. Angles

- On the square grid: Pick two adjacent dots and draw the line segment connecting them. Without using a protractor, draw the following angles at any end of the line segment: 45°, 135°, 225°
- On the isometric grid: Pick two adjacent dots and connect them with a line segment. Draw these angles at any end of the line segment: 30°, 60°, 90°, 120°, 150°, 210°

2. Collinear points

- Pick two points at random. Find a 3rd point that is collinear with them
- Verify collinearity with a scale. Can you prove collinearity? How?
- Repeat with other pairs of points on both the square and the isometric grids

3. Complete the rectangles and squares (check Figure A on Page 2)

- You are given two sides of a rectangle. Can you complete it?
- You are given one side of a square. Can you complete it? Is there only one square that you can draw using this line?
- Can you draw a square on the isometric grid if you are given one side? Why?

4. Parallel lines

- Pick any two points a bit far apart, draw the line joining them and pick a 3rd point not on the drawn line
- Draw a 2nd line through the 3rd point and parallel to the 1st line
- Justify that they are parallel

5. Perpendicular lines

Version 1	Version 2
Pick any two points a bit far apart, draw the line joining them and pick a 3rd point on the drawn line Draw a 2nd line through the 3rd point and perpendicular to the 1st line Verify that they are perpendicular. How?	Pick any two points a bit far apart, draw the line joining them and pick a 3rd point not on the drawn line Draw a 2nd line through the 3rd point and perpendicular to the 1st line Verify that they are perpendicular. How?

6. Reflections

Version 1	Version 2
Draw a horizontal or vertical line (mirror) and a scalene triangle on one side Reflect the triangle on the line	Use a mirror that is neither horizontal nor vertical

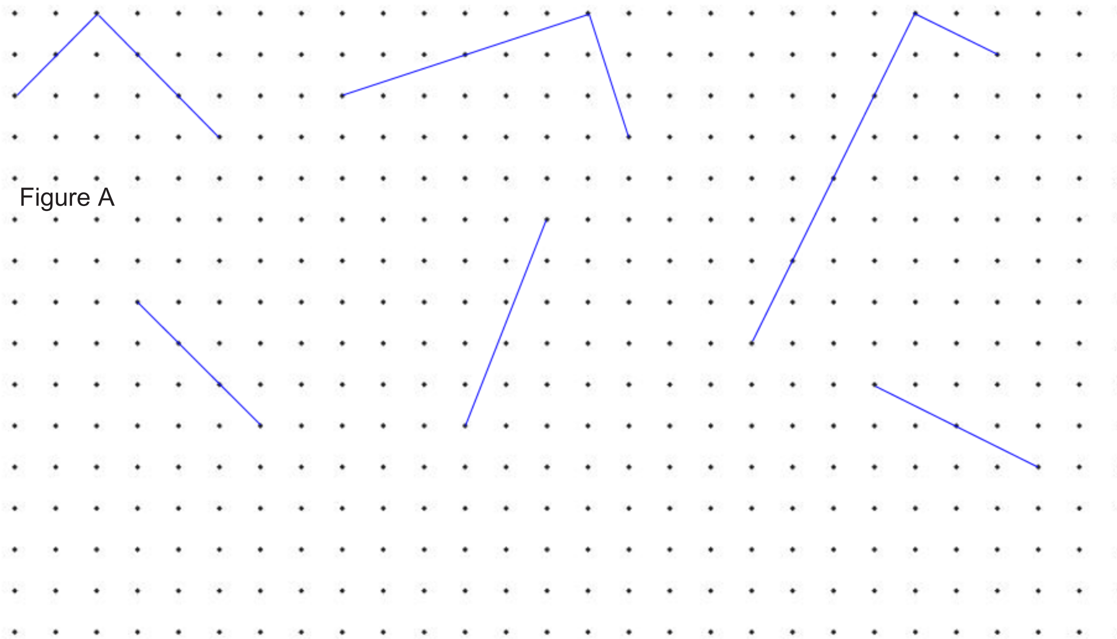
7. Rotations: Pick a point in the middle of the sheet and draw a scalene triangle

Square Grid	Isometric Grid
Rotate the triangle counter-clockwise by 90° and then by 180°	Rotate the triangle counter-clockwise by 60° and then by 120°

8. Double reflections (check Figures on Page 2)

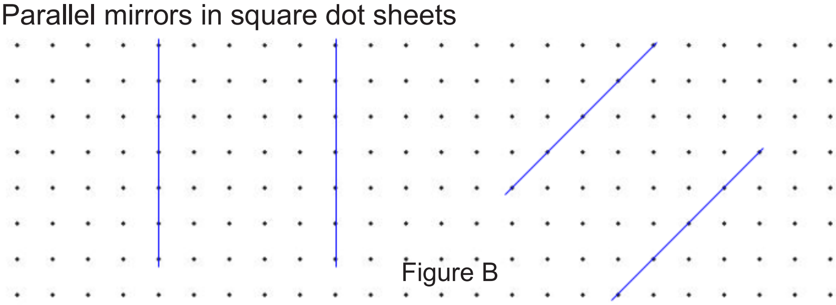
On parallel lines	On intersecting lines
Mirrors are a pair of horizontal or vertical grid lines (Figure B) Mirrors are a pair of (45°) slant parallel lines (Figure B)	On square grid: mirrors intersecting at 45° or 90° (Figure C) On isometric grid: mirrors intersecting at 30° or 60° (Figure D)

Complete the rectangles (2 sides given) and the squares (1 side given)

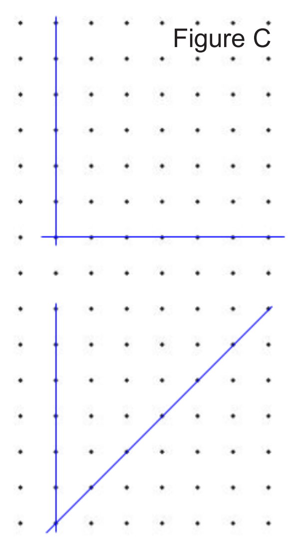


Use the following examples for pairs of mirrors for double reflection

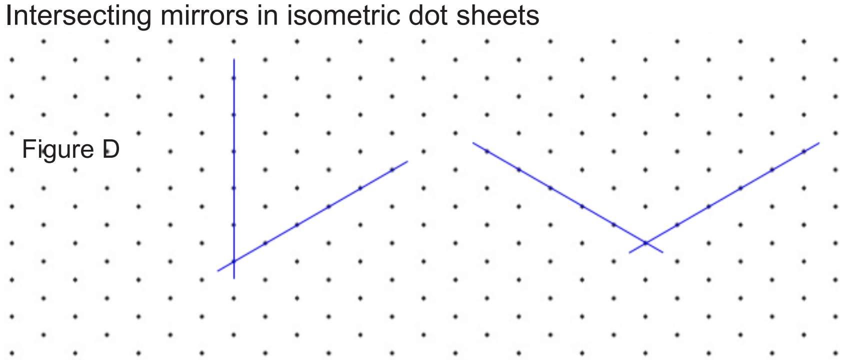
Parallel mirrors in square dot sheets



Intersecting mirrors in square dot sheets



Intersecting mirrors in isometric dot sheets



Exploring Spatial Understanding and Geometry on Square-Grids and Dot Sheets

The following activities can be done on square grids (Figure 1) and on rectangular dot sheets (Figure 2). These pave the way for more rigorous navigation of the Cartesian plane in the higher classes. Henceforth square grid will refer to both the actual square grid with lines as well as the rectangular dot sheets. [The advantage of rectangular dot sheets over the square grid from notebooks is that they do not have any lines.]

In general, one can start any of the activities on the square grid. Later, they should be tried on the isometric grid (Figure 3) as a challenge. Some activities should be done only on the isometric grid as indicated below.

Figure 1

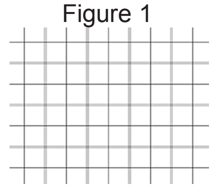


Figure 2

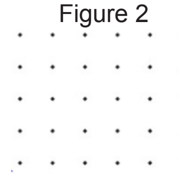
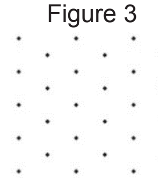


Figure 3



Materials required (other than dot sheets) will be scale, pencil, eraser and sharpener. It might help to have a protractor but that is necessary only for verification. Whenever a line has to be drawn, two grid points must be identified through which this line passes.

The activities are broadly in two categories: **A.** Drawing lines parallel to or inclined at a given angle to a given line **B.** Reflecting and rotating shapes

The topics which can be introduced or practised with these activities are Understanding Elementary Shapes (Class 6 NCERT Curriculum) and Symmetry (Class 7 NCERT Curriculum). However, with skillful facilitation, students can go far beyond these topics. These activities can be done with classes 5-8; the level of responses will of course depend on the topics that the students are familiar with. In many cases, students are asked to justify their answers and this will help them to develop their mathematical reasoning. This is also an opportunity for the teacher to facilitate their appreciation of mathematical rigor.

Throughout we have used *blue* for what is given and *pink* for what a child is supposed to do.

1. Angles - The features of the dot sheets should be utilized for drawing these angles. Children should be able to identify the square and equilateral triangle tiling in the respective dot sheets. Using the angles of these regular polygons they should be able to justify the presence of 90° and 60° angles. Halving these would generate 45° and 30° respectively which can be combined with the earlier angles to get the rest.

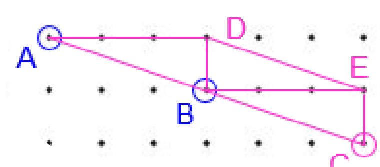


Figure 4

2. Collinear points If two consecutive grid points are selected, then finding a 3rd point would be too simple. So care must be taken to pick points which are further apart and not on the same grid line. The easiest way to find a 3rd collinear point is to mimic the path from the 1st point to the 2nd one to go to the 3rd from the 2nd. E.g. in Figure 4, A and B are the given points. The path from A to B and from B to C is '3 right and 1 down'. To prove that A, B and C are collinear, we can show $\angle ABC = 180°$ by observing that ΔABD, ΔEDB and ΔBCE are congruent (Why?) and using the angle sum property of a triangle. This '3 right 1 down' provides a beginning into 'run and rise' whose quotient (i.e., rise over run) is slope. Children should be able to eye-estimate and do double, i.e., 6 right and 2 down, triple, etc. If the given run and rise are not coprime, they should be able to identify grid point(s) within the line segment AB.

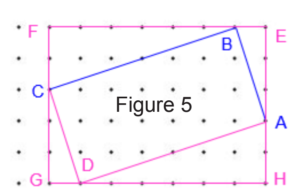


Figure 5

3. Complete the rectangles and squares - Whenever the given sides are at 45° slant, it is easier since mirror reflection can be used. However for the rest, justification can be provided with the help of congruent right triangles. E.g. in Figure 5, the same path is

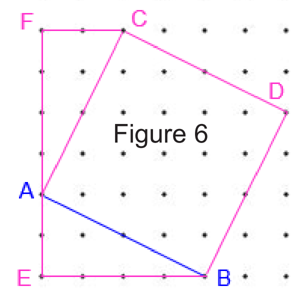


Figure 6

followed from C to D as in B to A resulting in $\Delta ABE \cong \Delta CDG$. So AB = CD and similarly BC = AD i.e. ABCD is a parallelogram with $\angle B = 90°$ (given) making it a rectangle. For square, the right angle needs to be constructed and that can be done on either side of the given line. E.g. in Figure 6 – the path A to B: 2 down 4 right changes to 4 up 2 right for A to C. This leads to rotation of ΔABE to ΔCAF and contributes to the $m \cdot m' = -1$ for slopes of perpendicular lines.

4. Parallel lines - Once again, the easiest is to follow the path from A to B and mimic that to go from C to D. That results in AB = CD and AB || CD. This can also be achieved by mimicking A to C i.e. 2 down 1 right for B to D as shown in Figure 7. Both can be thought of as a translation resulting in parallel lines.

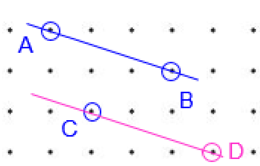


Figure 7

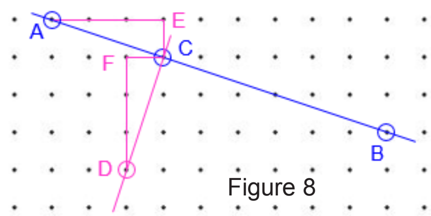


Figure 8

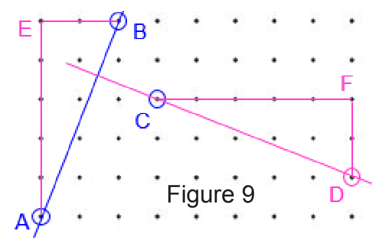


Figure 9

5. Perpendicular lines - It boils down to rotating a right triangle by 90°. E.g. in Figure 8, ΔACE is rotated to get ΔDCF while in Figure 9, ΔABE has been rotated to get ΔCDF. Both involves an exchange of run and rise and interchange of up and down.

6. Reflections - As a precursor to reflecting triangles (or any other shape), children should first reflect points on a line. Eye-estimation should suffice if they understand the properties of reflection. In particular, that if A' is the reflected image of A on the line PQ, then PQ is the perpendicular bisector of AA'. So to reflect a triangle, each of the vertices has to be reflected. Scalene triangles help in identifying which vertex got reflected to which one.

Various properties of reflection can and should be discussed after this activity. This includes the change in orientation, congruency of image and pre-image as well as image and pre-image being equidistant from the mirror. For version 2: the mirror can be at 45° with grid lines on square grid and at 30° on isometric grid.

7. Rotations - Similarly, rotating a point about another point should be tried first. Children should be able to eye-estimate and understand that if A' is the rotated image of a point A rotated by θ about another point O then $\angle AOA' = \theta$. Various properties of rotation can and should be discussed, especially that orientation remains the same.

8. Double reflection - This is an interesting exercise to observe that double reflection on parallel lines results in translation while that in intersecting lines generates a rotation. In addition, it is worth noting that the distance between image and translated pre-image is double the perpendicular gap between the parallel mirrors (Figure 10). This can be easily justified with the help of the image in between. Similarly it can be observed that the angle of rotation is double the angle between the intersecting mirrors. Proof again utilizes the in between image (Figure 11).

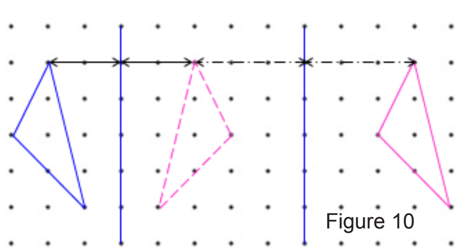


Figure 10

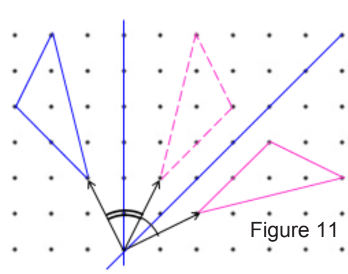


Figure 11