At Right Angles

A Resource for School Mathematics

INTRODUCTION TO ALGEBRA - II

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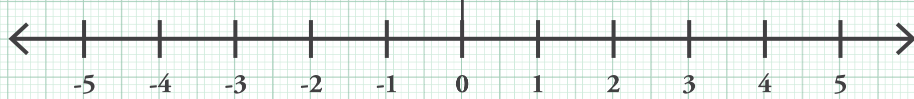

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INTRODUCTION

This article is Part II of the series 'Algebra – a language of patterns and designs.' The approach is based on the perception of algebra as a generalisation of relationships.

In Part I [http://teachersofindia.org/en/article/introduction-algebra-right-angles-pullout], we introduced the ideas of *variable*, *constant*, *term* and *expression* via numerical patterns. Various operations (addition, subtraction, multiplication) involving terms and expressions were also studied.

Now in Part II, we revisit the usage of words such as *variable*, *term* and *expression* and concepts and operations involving terms and expressions in the context of ***geometric designs*** (line designs, 2-D designs, 3-D designs).

Geometric designs are seen everywhere. Tile designs on floors, brick or stone work on walls, partitions of windows and doors, cardboard boxes (toothpaste boxes, soap boxes …) and many other everyday objects can be described in algebraic form.

As always, we begin with familiar concrete objects and use algebraic language to describe them before moving progressively to abstract algebraic expressions.

Ideally students should be exposed to algebra initially through the pattern approach followed by the design approach. However, the two approaches are independent of each other.

By approaching algebra through different routes, we will be able to make a robust link between informal algebra at the primary stage to the more formal algebra which students encounter later. Also it will facilitate students' fluency in the language of algebra, i.e., understanding variables and symbols and being able to use algebraic rules correctly.

Prior knowledge: Students need to be familiar with notions such as line segment, length, region and area of a rectangle, area of a square, capacity, volume of a cuboid and volume of a cube.

Keywords: Algebra, language, pattern, geometric design, variable, constant, term, expression, operation.

ACTIVITY 1

Objective: Introduction to design language (in the context of line designs) and the usage of variables for different lengths.

Materials: Sets of straws or straight sticks of different lengths. Dot paper

A few examples of real life situations:

How do we describe this ladder?

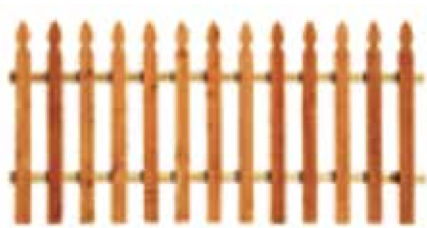
The ladder is made up of some long sticks and some short sticks.

If we take the ***length*** of the long stick as ***'l units'*** and the *length* of the short stick as ***'s units'***, we can describe the ladder design as 2*l* and 5*s* or **2*l*** + **5*s***.

How do we describe this house?
We can express the house design as 5*l* and 3*s*.

We can also express it as **5*a*+3*b*** (where a stands for the length of the long stick and b stands for the length of the short stick).

How do we describe this fence?



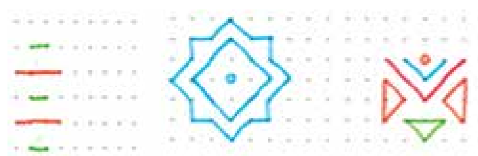
How many *l*'s? How many *s*'s? Here *l* is the length of the long stick and *s* is the length of the short stick.

Initially students can make designs using two different lengths and describe it as an expression.

Later on they can make designs using three or four different lengths.

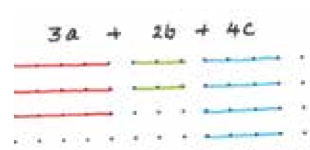
Here is a design made of three different lengths. It can be expressed as 3*a* + 2*b* + 4*c*.

Students can now be given designs of a similar kind to describe using appropriate design language.

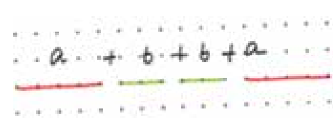


Set them exercises of the reverse type as well. Give students dot paper and ask them to create designs for some given expressions.

Example: 3*a* + 2*b* + 4*c*



Example: *a* + *b* + *b* + *a*



ACTIVITY 2

Objective: Addition and subtraction of expressions through line designs

Materials: Straws or sticks, Dot paper

Ask the first student to build a design to show a given expression, say, $4p + 3q$.

Ask the second student to show $2p + 1q$

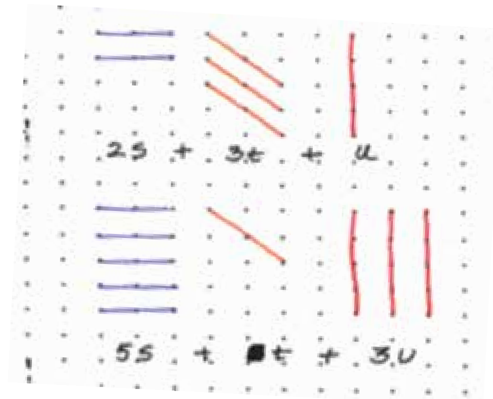
Verify that the second student has chosen the straws that correspond to the lengths p and q chosen by the first student. If not, it is an opportunity for discussing that p and q stand for specific lengths and line segments of the same length will be represented by the same letter. The students need to understand that different variables represent different numbers.

Now, how do we read these two designs together? They will be read as $6p + 4q$.

Ask students to record the design and expressions in a dot paper to show addition of expressions.

Give students a few more sets of expressions for which they can make corresponding drawings and sum them.

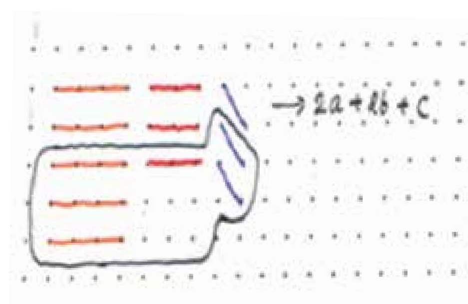
Example: $2s + 3t + u$, $5s + t + 3u$



In a similar manner one can demonstrate subtraction of expressions. Lay out the design for an expression, say, $5a + 3b + 3c$.

Circle the sticks to be removed. Say, $3a + b + 2c$.

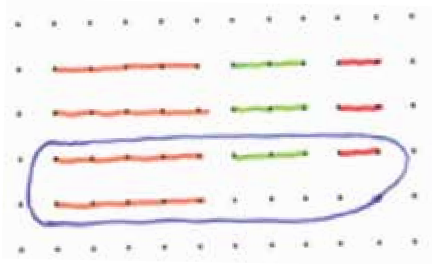
What is left?



$2a + 2b + c$.

This operation can be recorded on the dot paper as shown.

Give students some line segment designs as shown here to record the expressions and the subtraction of the given expressions.



They will give the expression for the initial design set up, then for the removed set (lines enclosed within loop are to be removed) and finally for the remaining set.

Note: Since these are concrete examples the teacher cannot, as yet, give examples of the following kind: Subtract $a + c$ from $2a + 3b$.

Give students a few more sets of expressions for which they can make corresponding drawings, subtract them and give the expression for what is left, like the following:

- Set up a line segment design to show subtraction of $4a + 2b + c$ from $7a + 5b + 3c$.
- Set up a line segment design to show subtraction of $5c + 8d + e$ from $10c + 8d + 3e + f$.

ACTIVITY 3

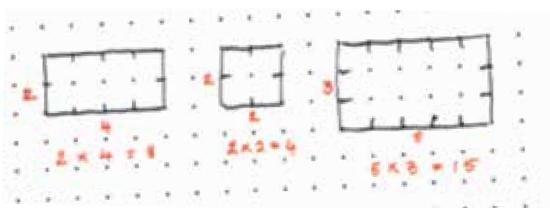
Objective: Design language for plane regions involving two variable terms.

Materials: Rectangles of different sizes (collection of visiting cards and greeting cards will help)

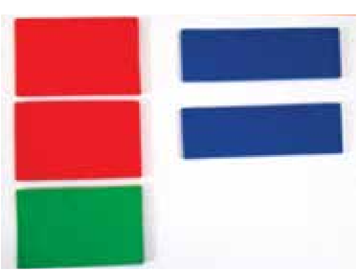
Prior knowledge: Familiarity with area formula for rectangles and squares.

Here, again, it is good to start using examples from real life situations before moving into abstract designs.

Revise the concept of area of rectangle and square using grid shapes as shown.



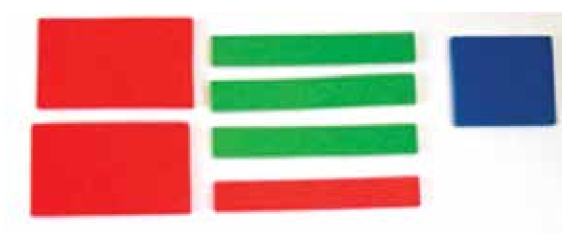
Set up a design with rectangles of two different sizes.



The length and breadth of the rectangles can be named using variables.

How do we describe this set up?
It would be $3ab + 2cd$.

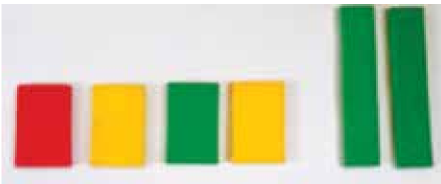
What will be the expression for this set up?



It would be $2ab + 4ef + gh$.

It is possible that different rectangles may have one common edge. Discuss the need for the usage of the same variable in such situations. This possibility is taken up in the next activity.

Give students some plane region designs as shown to draw in the dot paper and record the expressions.



Similarly give some expressions for which they need to make corresponding plane designs,

Example:

$3ab + 2pq + mn$,

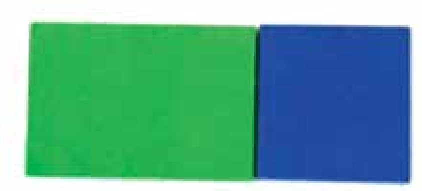
$2a^2 + 3b^2 + c^2$,

$ab + a^2 + b^2$.

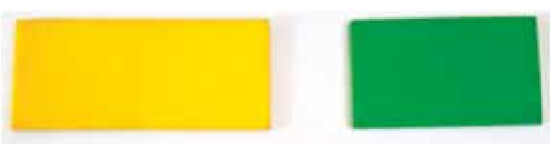
ACTIVITY 4

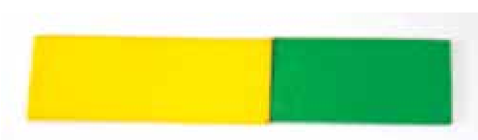
Objective: Design language for composite figures.

Materials: Rectangles and squares of different sizes which have one edge in common. Dot paper



When two rectangles or a rectangle and a square have one edge in common they can be combined to form a composite figure as shown.





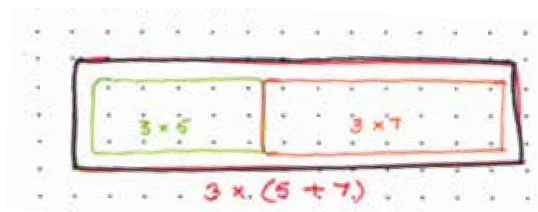
Note for the teacher: This will give rise to multiplication of a two term expression by a single term (binomial by a monomial).

Teacher can first place the shapes separately and state the design language as $ab + ac$.

Now the shapes can be brought together and stated as $a(b + c)$.

Through this, the law $ab + ac = a(b + c)$ (i.e., the distributive law) is established.

This can be further reinforced by assigning numerical values to the variable and demonstrated through dot arrays as shown.

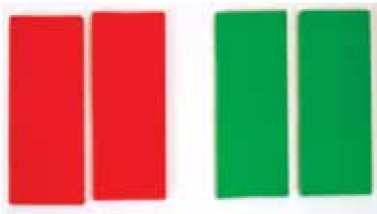


$$3 \times 5 + 3 \times 7 = 3 \times (5 + 7)$$

What will be the expression for this set up?

It would be $jk + jk + jk = j(k + k + k) = 3jk$.

What is the expression for this set up?



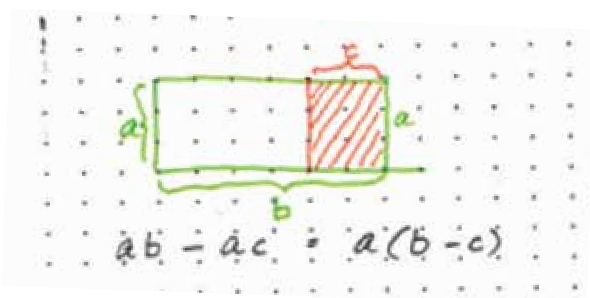
$2ab + 2ab = 4ab$.

Give students some plane region designs as shown above to draw in the dot paper and record the expressions.

Similarly give some expressions, as given here, for which they need to make corresponding plane designs and write the expression for the composite figure.

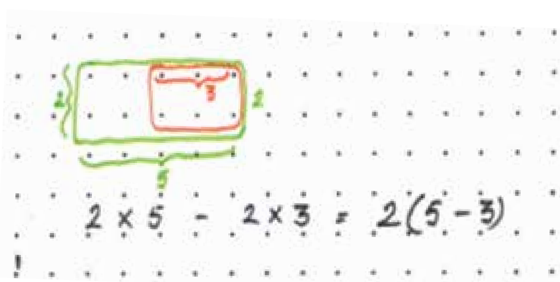
$$kl + l^2, \qquad 3pq + 2pr + p^2$$

At this point teacher can also discuss the design for $ab - ac$. It would look like the following.

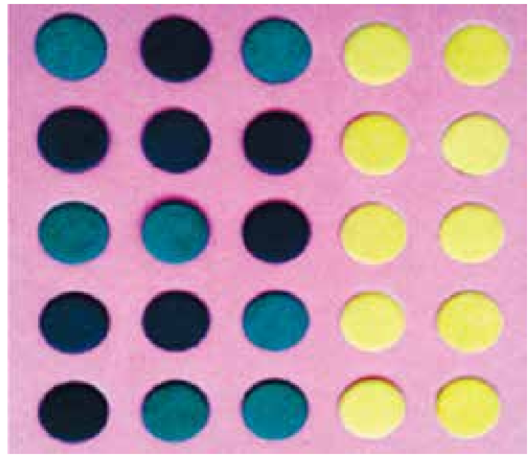


Again it can be established that $ab - ac = a(b - c)$.

This can be further reinforced by assigning numerical values to the variable and demonstrated through dot arrays as shown.



$2 \times 5 - 2 \times 3 = 2 \times (5 - 3)$



$5 \times 5 - 5 \times 2 = 5 \times (5 - 2)$

A few more such examples can be discussed.

$pq + pr + ps$

$ab + cb + db + fb$

Ask students to show using dot paper:

$a(b + c + d) = ab + ac + ad$.

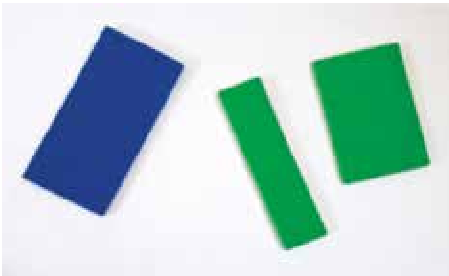
$p^2 + pq + pr = p(p + q + r)$.

ACTIVITY 5

Objective: Rules about addition, subtraction of like and unlike terms, Multiplication of expressions by a single variable

Materials: Rectangles and squares of different sizes. Dot paper

Vocabulary: Like rectangles, unlike rectangles, Like terms, unlike terms



Ask students to pick up any two rectangles. Pose the question 'are these rectangles alike?' or 'are these rectangles different?'

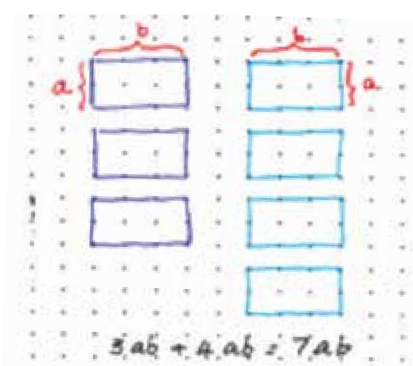
If the two rectangles have the same length and breadth they are like rectangles.

Students may pick up two rectangles which have the same length but a different breadth or which have the same breadth but different lengths or which have different lengths and different breadths.

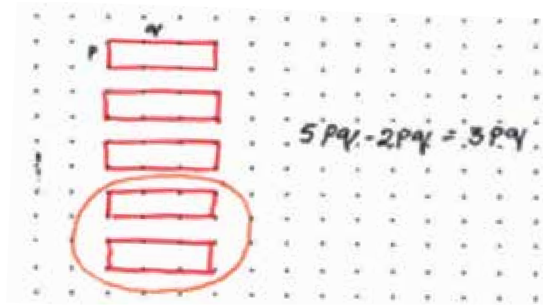
How do we describe them?

If two rectangles differ either in length or breadth or both they are unlike rectangles.

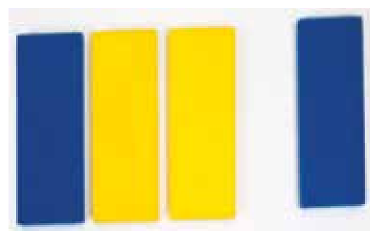
Show them that like rectangles are referred to using like terms and that like terms can be added only to like terms. Similarly like terms can be subtracted only from like terms.



Example: $3ab + 4ab = 7ab$



$5pq - 2pq = 3pq$



$4cd - cd = 3cd$

Also show them that unlike rectangles are referred to by using unlike terms and unlike terms cannot be added or subtracted.

$4xy + 2ab$

Let students draw figures in the dot paper to demonstrate the following.

$ab + ab + ab = 3ab$.
$3mn + 5rs - mn - 3rs = 2mn + 2rs$.

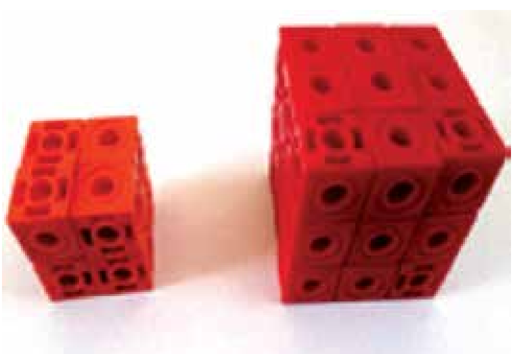
Students will now be in a position to make the transition to handle addition and subtraction of sets of abstract expressions of the following type:

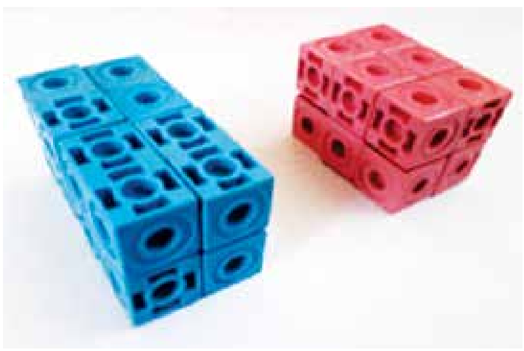
Add:	**Subtract:**	**Multiply:**
$2ab + 3cd + ef$	$5ab + 4cd + fg$	$a + 2b + c$ with d
$ab + 2cd + ef$	$ab + 3cd + fg$	
$4a^2 + 6b^2 + c^2$		
$2a^2 + b^2 + 3c^2$		

ACTIVITY 6

Objective: Volume

Materials: unit cubes, triangular. Dot paper





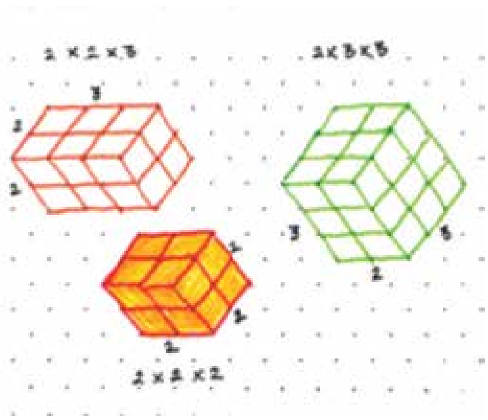
Initially let students build cubes and cuboids of different sizes.

Let them note down the dimensions in a table format (length, breadth, height, volume) to discover the formula for the volume of a cube or a cuboid.

They can fill the volume column by counting the number of cubes and noticing the relationship between the length, breadth and height.

Since the volume of 3D blocks is given in terms of length, breadth and height, the design language involves the use of the product of three variables.

Students can record these on triangular dot paper (also called isometric dot paper).



ACTIVITY 7

Objective: Design language for sets of blocks

Materials: Cuboids and cubes of different sizes and same size. Cardboard boxes of the same kind (soap boxes, toothpaste boxes)

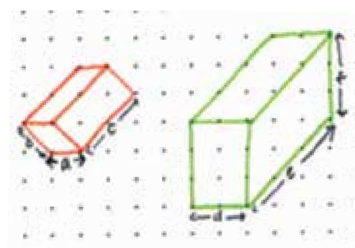
Prior knowledge: Volume of cube and cuboid
Vocabulary: Like terms, unlike terms

Show that cuboids having same length, breadth and height are referred to by like terms as shown.

Teacher can place different combinations of boxes and get the children to describe the set up using design language.

Example: $3abc + 2def$

Sides of unlike cuboids will be indicated by different letters (variables) and are referred to by unlike terms.

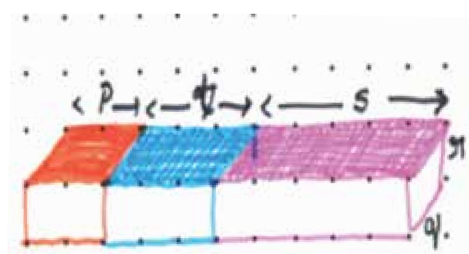


ACTIVITY 8

Objective: Design language for combined blocks. Multiplication and factorisation

Materials: Cuboids and cubes which have common faces can be brought together. Design language for them can be given initially considering the blocks separately and then as combined.

$abc + abc + abc = 3abc$



Show that $pqr + tqr + sqr = (p + t + s)\, qr$

At this point students should now be ready for addition and subtraction of abstract expressions of the regular kind. **For example:**

Add:	**Subtract:**	**Multiply:**	**Factorise:**
$abc + 2def$ $3abc + 4def$ $2a^2b + b^2c + c^2a$ $3a^2b + 4b^2c + c^2a$ $a^2b + 2b^2c + c^2a$	$5pqr + 3uvw$ $pqr + 2uvw$ $7a^2b + a^2c + 3a^2d$ $5a^2b + a^2c$	$p(p^2 + pq + pr + qr)$	$a^3 + a^2 b + a^2c$

CONCLUSION

By the time we complete the activities suggested in Part I and Part II in this series, students should feel comfortable in the use of algebraic concepts and words such as *variable*, *term* and *expression,* and in performing various operations using them.

They should be in a position to manipulate similar abstract expressions. Once the general principles of 'like' and 'unlike' terms have been understood and the rules of operations internalized, students should be ready for the take-off stage. They should be able to use the same laws and principles with higher powers and multi-variable terms and expressions.

Part III of this series will be on *approaches to equations.*

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