Shapes with Sutli: The String Game

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This article describes an activity where students created different geometrical shapes using a closed-loop string and developed conceptual understanding by engaging with properties of the shapes. The activity encouraged them to think deeply about the meaning of points, straight lines, edges, faces, and angles of geometrical shapes. Using standard models which are generally available, students only get to view geometrical shapes or build them by following a set of instructions. However, the activity described here provided students the opportunity to interact with each other, build various shapes and think creatively about how to prove their properties, thus, developing an in-depth understanding.

In many classrooms, Mathematics is taught as an abstract subject with emphasis on developing procedural knowledge. This renders Mathematics as a dry and difficult subject for most students. They develop a fear of the subject and prefer to drop it at the earliest possible opportunity. The way students engage with Mathematics is neither perceptibly useful nor interesting. However, if they are engaged in meaningful activities, which enable them to explore and visualize concepts, Mathematics can become a far more interesting subject.

Many students begin to dread the subject when they encounter theorems and proofs, the basic pillars of logic and reason. In the school curriculum, it is in the topic of geometry that the student is introduced to notions of argumentation and proof for the first time. Before working with proof, students need to visualize different shapes and their properties and also strengthen their understanding of the basic definition of shapes. This article describes a classroom activity where students of grade 9 constructed various two-dimensional and three-dimensional shapes while justifying and proving their properties at the same time. It is based on the old childhood game of "Sutli" which many of us may recall, and converts it into a Math game. [For the benefit of those who haven't heard of this game, it is played with a sutli or string, looped around the players' fingers to make patterns. It can be played by a single player and also in pairs by making patterns with their fingers. One partner makes a pattern,

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the other builds on it, and the sutli keeps changing hands from one partner to the next till they get stuck, and can't make any more, or someone makes a mistake and the game has to be started all over again.]

In a traditional mathematics classroom, shapes are usually introduced through drawings on the blackboard. In some schools, students are shown 2D and 3D models of different geometrical shapes. A common hands-on activity entails cutting out and building a shape using a net. Such tasks, although interesting, allow students to create shapes by following a set of instructions, but do not help to develop geometrical reasoning related to the shapes.

The Activity

The objective of this activity is to build several 2D and 3D shapes and justify their properties through logical arguments [1]. The framework proposed by Cathy Humphreys [2] of *convince yourself, convince a friend, and then convince a sceptic* was used for justification. The activity was tried out with a class of 30 students where they worked in groups of about four. They were given 90 minutes to complete the task. Each group was given an 8 feet long closed-loop string and was required to build a given shape with the loop. The conditions posed were as follows:

a. The string knot cannot be opened at any time,

b. every person in the group must have at least one hand on the string,

c. the group must use the entire string,

d. the group must be able to justify their claim, and

e. no instrument from the geometry box may be used to build/measure the shape.

The shapes chosen for the building exercise were square, pentagon, cube, tetrahedron, square-based pyramid, and octahedron.

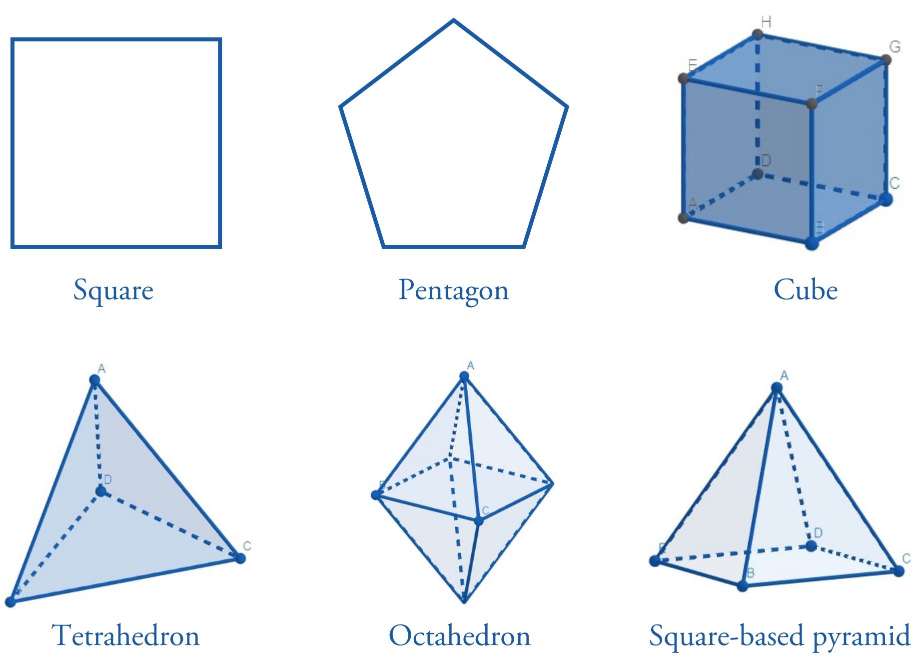

Figure 1: Shapes to be created by the closed loop string

Discussion

Several methods were devised by students to build each shape. In this article we will discuss the methods for the square and cube. While the students attempted the task, the teacher facilitated the process by asking thought provoking questions. At the start of the activity, the teacher and support staff demonstrated the making of an equilateral triangle using the closed loop string and played the role of a friend and a sceptic. Some of the questions posed during this process were: how do you know it's a triangle? How can you prove all sides to be equal? What can you say about the angles? Why? In general, the teacher acted as the sceptic and 'pushed' the students to see and formulate multiple paths of thinking and justify their arguments regarding the shapes.

A. Building a Square

Different methods for building a square were implemented. Some of them are discussed below:

a. The loop was folded to get four equal sides and was opened up to form a quadrilateral (Figure 1). The interior angles were adjusted so that all were of equal measure (right angles). Some groups tried to prove that all angles are 90 degrees, while the others tried to prove that all angles are equal. To prove that a given angle was a right angle, students used the corner of a book or corner of wall and floor as a reference. The challenge in this approach was – how does one prove the corner of a book to be a right angle? The justification used was – if all the interior angles match the corner of the book, this would mean they are all equal, hence the quadrilateral would be square.

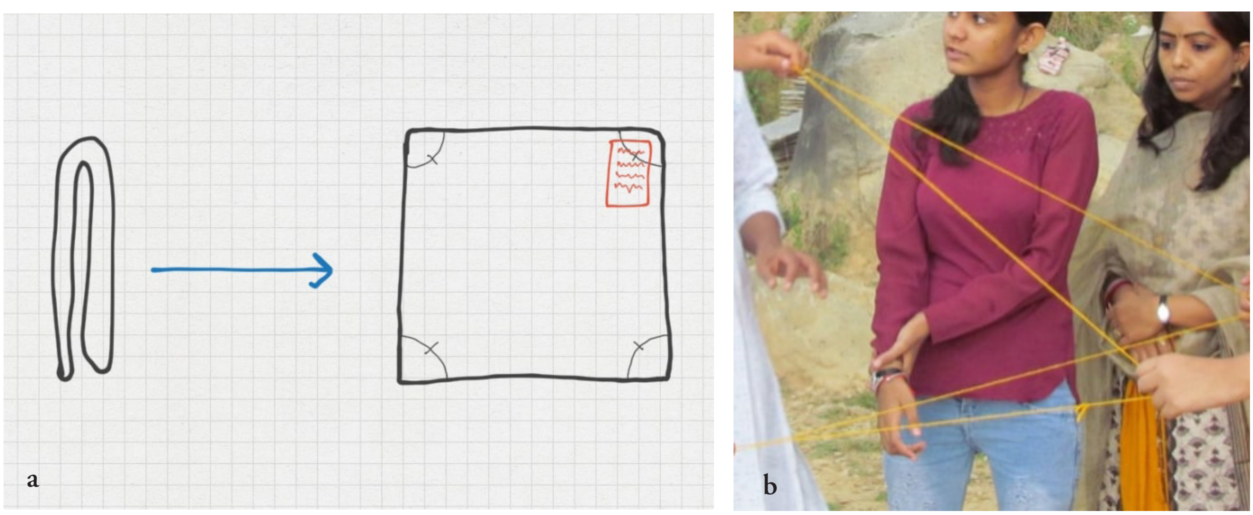

Figure 2: (a) Equally folded loop converted to square, angle measured with a book corner. (b) Students working on building a square

b. The loop was folded in such a way that a quadrilateral was formed with both its diagonals (Figure 3). All sides were compared to ensure their equality and both diagonals were also shown to be equal in order to justify that the quadrilateral was indeed a square. Some groups did not make the diagonal, but measured it with a stick.

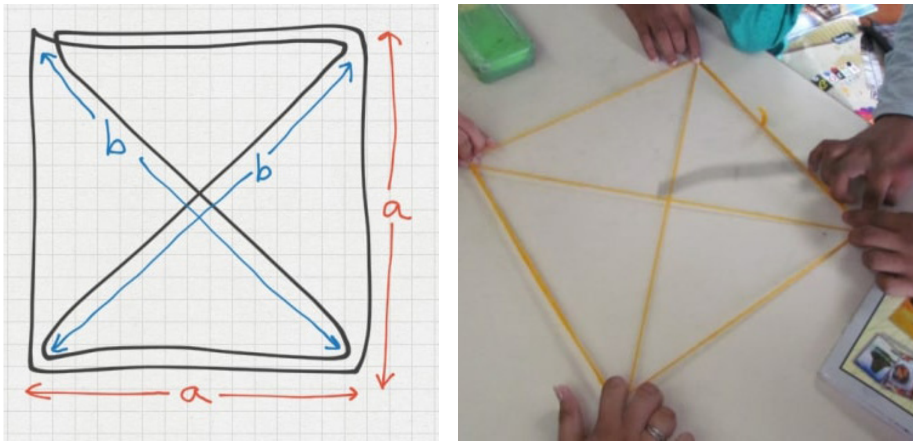

Figure 3a. Folding strategy for case (b)

Figure 3b. Students measuring the diagonal with a stick

c. In this method the loop was folded equally twice to get eight equal sides as shown in figure 4 below. Students held the centre point (mid-point of w shape in figure 4), and extended the edges to form a plus sign. All four angles of the plus sign were shown to be equal using the corner of a book. Finally the four central points of the thread were flipped to form the square.

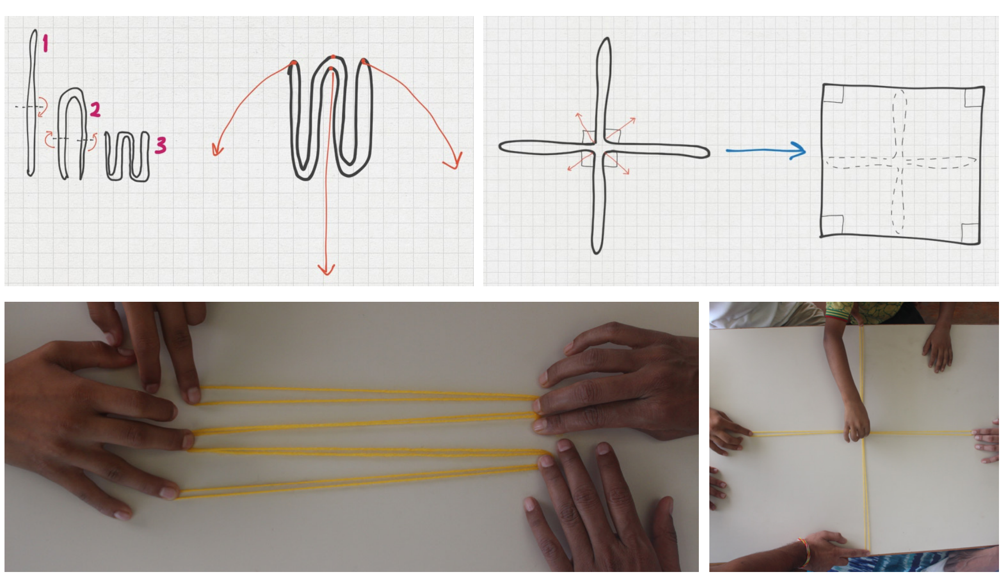

Figure 4: Folding strategy for case (c)

A critical point arose in the discussion - it is important to prove that all the four points are in the same plane. In general the students found it easy to prove the sides to be equal, but proving the angles to be equal was a bit challenging for most groups. The teacher guided the students by posing the following questions: What does it mean to have a right angle? Where do we see it around us? If all angles are 90 degrees, how do they compare with each other? If the student responds that it means all are equal, then how can you prove they are equal? What strategy can be used for this comparison?

One group also discussed the idea of making a separate knotted loop of Pythagorean triplet (3, 4, 5) to prove the corners to be right angles (as shown in Figure 5).

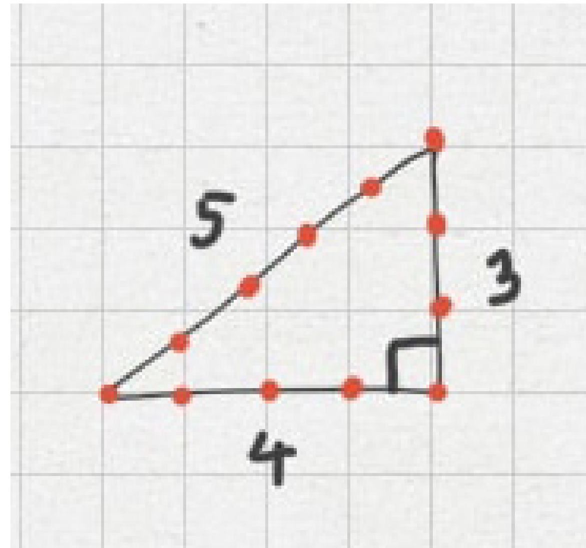

Figure 5: A closed loop with 12 equidistant knots being held as a right angled triangle

By the end of the exercise, the groups had developed the basic definition of a square. They articulated that 'A square is a quadrilateral in which all sides are equal and all angles are equal.' One key understanding that developed was, proving all interior angles to be equal is different from proving that an angle is 90°. To prove that a shape is a square, the following conditions must be met:

a. All the points are in the same plane, and all four edges are straight lines,

b. All sides are equal, and

c. One of the following:

    i. All interior corner angles are equal,

    ii. All interior corner angles are right angles,

    iii. The two diagonals are equal.

The groups were intentionally not probed about defining every shape before the building exercise. Initially, the gap in the definition of a square did not come out when they convinced each other in their groups. But when the facilitators came and questioned as sceptics, they realized what they were missing in their proof of a square. By the time the cube was presented to them, they were already asking the sceptic questions.

B. Building a Cube

There was an initial sense of disbelief when students were asked to build a cube with the loop string, but they quickly got to the task. One key element of the construction was having a clear understanding that for any face to be a square, all the points must lie on the same plane. It helped to draw the perspective view of the cube (as shown in Figure 1) on paper before starting to build the cube. It was important to understand the definition of a cube, and what was needed to prove that the structure was a cube. The importance of precise mathematical terms was apparent for a productive argument.

Some of the methods for building a cube were as follows:

a. **Loop method:** Open up the cube faces and visualize the cube net. Follow steps 1 to 3 as shown in Figure 6. Adjust each loop in the square shape as shown in step 4. Same coloured dots represent the corners of the squares meeting at the same vertex. Once all the vertices are connected a cuboid will be formed. Sides need to be adjusted for square faces.

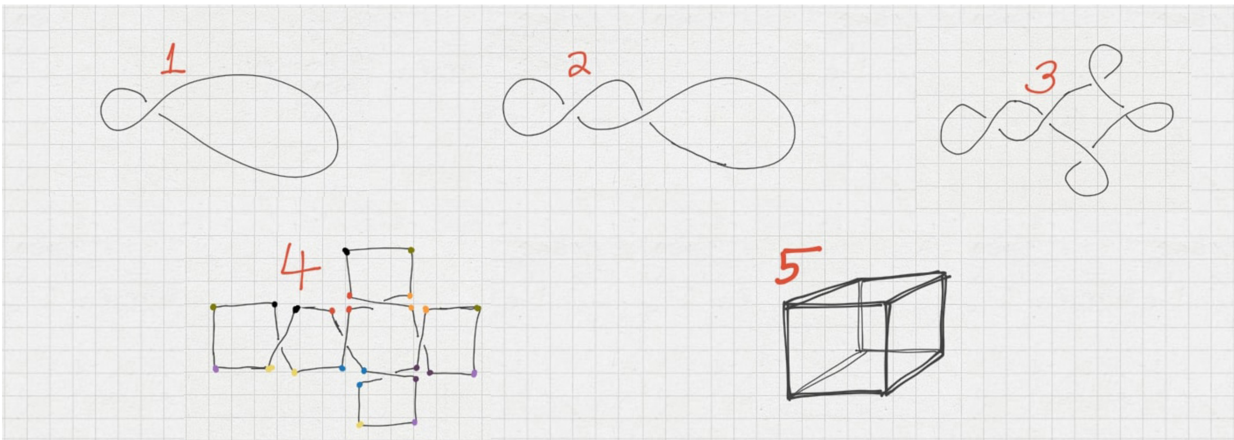

Figure 6. Loop method

b. **Pinch-and-Extend Method:** Make a square face on a flat surface, pinch a corner to make a small loop. Pull the small loop away from the square to make one edge. Keep the original square shape, but let it shrink. Do this to all four corners of the original square, and pull the new four edges vertically up from the square surface. There are still four edges of the cube remaining. Further extend and bend each loop edge at right angle filling in these four missing edges.

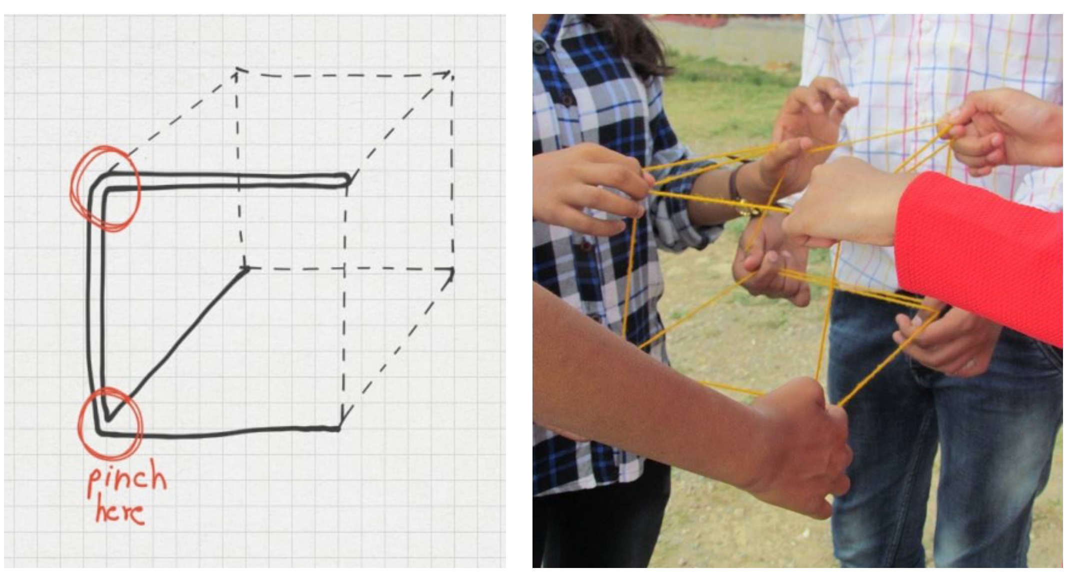

Figure 7: Pinch-and-Extend method

c. **Trace-the-Edge method:** Keep tracing all the edges of the virtual cube and come back to the starting point. Hold all the vertices and adjust to make all faces square-shaped to build the cube.

Proving the 3D design to be a cube shape was a challenge for many. The conceptual clarity regarding the properties of a cube was the key. Many groups struggled with the thought that they needed to prove each face to be a square, and that all adjacent planes were perpendicular to each other. After much deliberation, the groups reached a definition: "A cube is a closed 3-dimensional structure with six square faces, with three faces meeting at each vertex." The minimum conditions needed to prove a cube shape using a string are:

a. The 3-dimensional shape must have an outline of six faces,

b. All faces outlined must be square shapes that are congruent to each other,

c. Three faces meet at each vertex.

There was further deliberation on what it meant to be a 'face.' Since a thread was used, outline of the face was constructed and not the face per se.

Students also created a tetrahedron and an octahedron with the loops (Figure 8). A similar process of building the shape and developing a proof along the way was followed. Some groups went further and started working on other shapes such as a dodecahedron at the end of the 90-minute session.

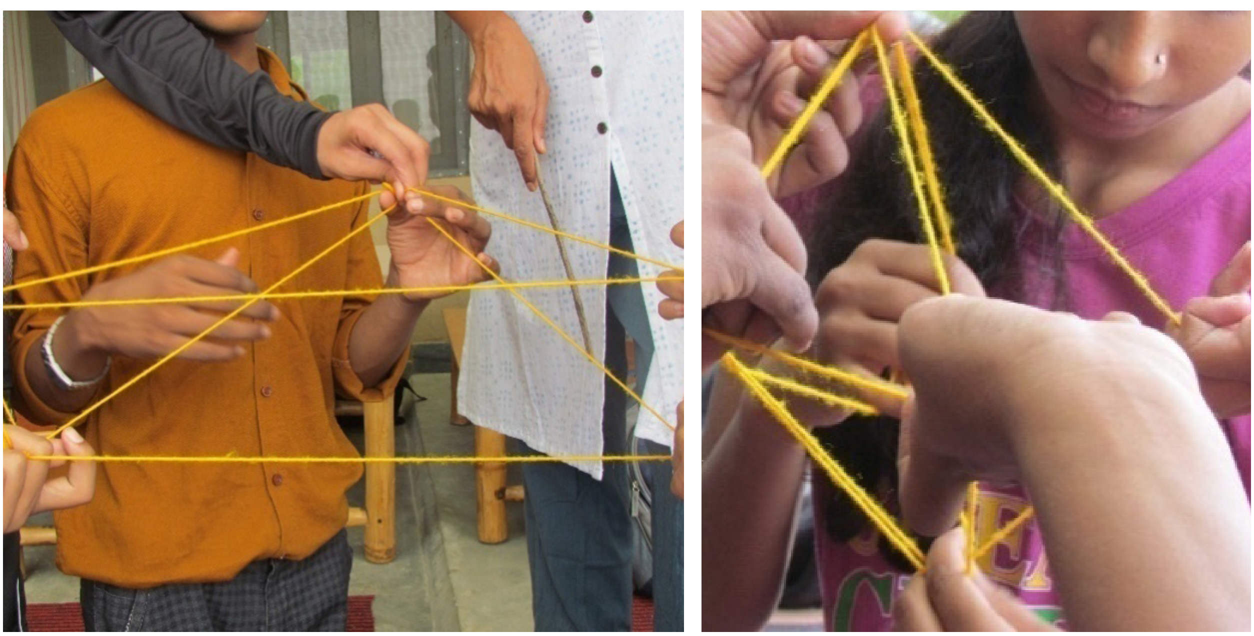

Figure 8: Students building square based prism and octahedron

The session facilitators played the role of the sceptic, encouraged students to think deeply about the meaning of a point, a straight line, an edge, a face, or an angle in the geometrical sense. Some students even argued about the meaning of a vertex while holding it with their fingers. The definitions of many geometrical terms were revisited, questioned and discussed. The importance of correct terminology and its mathematical meaning in a conversation became apparent. For example, it wasn't 'side' of a cube anymore, it was side of a square and edge of a cube; it wasn't being called 'corner' of a cube, it was a cube's 'vertex'; that in a pentagon all sides and interior angles need not be equal, only a regular pentagon will have that and so on. It was very satisfying to see that every group member engaged with the activity without any fear of failure. Students approached the task in multiple ways, listened to each other's ideas and collaborated in their work. Many students felt that they had developed a clear understanding of these shapes for the first time. Students who had initially defined square as a quadrilateral with equal sides, added 'all interior angles equal' to their definition. In the attempt to figure out the elements needed to prove a cube, the students thought about the angular relationship between not only two edges, but also between two planes. Students also realized that they didn't need to memorize the properties of the shapes, such as number of faces, edges, vertices, interior angles, etc., as they could now visualize the shape and work out their properties whenever required. The exploration of shapes did not stop with the session, but became a point of discussion even days after the session. This was indeed a high point of the activity.

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