W.W. Sawyer
The Universal Math Teacher

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As a corollary of Section 8 of the Right to Education Act, every young Indian citizen in elementary school today has the legal right to get mathematics education of good quality. Perhaps India is the only country in the world, where this is legally mandatory. Two questions are now squarely on the agenda of Indian math education: "What is math education of good quality?" and, "Is it possible to ensure this for every child?"

Prof. W.W. Sawyer, who passed away in the year 2008 at the age of 97, was perhaps the first person to articulate math universalization in the sense of the RTE Act and to engage in depth with both these questions.

In a 1958 article, titled 'The Possibility of Universal Mathematical Literacy' (TPUML), he wrote: *"We are facing a great crisis and a great opportunity. Changes in science and technology are tending to make semi-skilled labour obsolete. The demand for highly educated people, teachers, mathematicians, technicians is rapidly increasing. Social and political dislocation can only be avoided if we are able to educate to a high standard far more people than has previously been considered possible. The ability to think mathematically will have to be taken for granted much as the ability to read a newspaper is at present. Such a change will seem fantastic to many people. So would universal literacy have seemed absurd a few centuries ago.*

Two possible viewpoints on twentieth century education.

a. It represents a close approach to the best that is humanly possible.

b. It represents the first gropings of a new society. Universal schooling is hardly a century old. Its standard of efficiency may be compared to the standards of industrial efficiency of 1750.

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To enter a new stage of history is always difficult…

The population today divides sharply into those who hate and fear mathematics and a minority of mathematicians.

The remarkable thing is that such an outcome is accepted as normal. It is as if physical education cripples 90% of the children taking it."

The conviction that universalization of mathematics is possible, in the way that every citizen becomes comfortable with her mother tongue, is the foundation of Sawyer's work on math education that spanned almost 8 decades, all levels from elementary to university math and across several continents. His definition of success in mathematics, like all his writing, is simple but profound: *Complete success would mean that every individual felt, "I enjoyed the mathematics that I had time to learn. If I ever need or want to learn some more, I shall not be afraid to do so."*

Mathematics is commonly thought to be a difficult and esoteric subject, accessible only to a select and privileged few. Sawyer's entire lifework was dedicated to proving the converse – that ordinary people like you and me can be taught to understand, learn and enjoy mathematics.

In the following, we survey briefly some of his insights that are as relevant and pertinent today as when they were written many decades ago.

Sawyer points out that *rigour in the teaching of mathematics* should never be confused with the problem of *rigour in mathematics*. Though always gentle, he is unsparing in his criticism of the almost universal absence of rigour in math teaching. In 1946, Sawyer visited India and delivered a lecture to the Indian Mathematical Society titled "The Teaching of Mathematics." *Probably there is no subject which offers such possibilities for misunderstanding between teacher and pupil as mathematics does. The teacher stands at the blackboard. It is perfectly clear to him what the symbols mean, and what conclusion can be drawn from them. It is completely otherwise with many of the pupils. What the symbols are meant to represent, how the teacher knows what is right and what is wrong, what is the object of the whole business anyway – all this is wrapped in mystery. The great majority of students say to themselves,* ***"We shall never learn this stuff, but we want to get through the exam. We'll have to learn it by heart."***

This is not a satisfactory state of affairs. This learning by heart not only imposes a quite unnecessary strain on the student; it is also quite useless. It gives neither an understanding of the subject, nor the power to apply mathematics in ordinary life.

The more we can see things from the pupil's point of view, the better teachers we shall be. And the first question in the pupil's mind is, ***"Why do we have to do this at all?"*** *When I was at school, the boys were always asking this – and they never got a satisfactory answer. The teachers made up all kinds of answers, but they were none of them very convincing. The fact is, I think, that mathematics is taught because it is* ***the custom*** *to teach it.*

The same point is made, more sharply, in his article 'From Abstract to Concrete' (1962): *The depressing thing about arithmetic, badly taught, is that it destroys a child's intellect, and to some extent his integrity. Before they're taught arithmetic children won't give their assent to utter nonsense: afterwards, they will. Instead of looking at things and thinking about them, they make wild guesses in the hope of pleasing the teacher or an examiner.*

In his book *A Concrete Approach to Abstract Algebra* (1959) he writes about how not to teach: *"In planning such a course, a professor must make a choice. His aim may be to produce a perfect mathematical work of art, having every axiom stated, every conclusion drawn with flawless logic, the whole syllabus covered. This sounds excellent, but in practice the result is often that the class does not have the faintest idea of what is going on. Certain axioms are stated. How are these axioms chosen? Why do we consider these axioms rather than others? What is the subject about? What is its purpose? If these questions are left unanswered,*

students feel frustrated. Even though they follow every individual deduction, they cannot think effectively about the subject. The framework is lacking; students do not know where the subject fits in, and this has a paralyzing effect on the mind."

But there is an alternative:

"On the other hand, the professor may choose familiar topics as a starting point. The students collect material, work problems, observe regularities, frame hypotheses, discover and prove theorems for themselves. The work may not proceed so quickly; all topics may not be covered; the final outline may be jagged. But the student knows what he is doing and where he is going; he is secure in his mastery of the subject, strengthened in confidence of himself."

And what constitutes good math teaching?

The most important thing in the early teaching of mathematics is that the student should form the habit of weighing evidence, of deciding for himself. (Vision in Elementary Mathematics 1964)

The essential thing is to arouse interest in the subject:

I am convinced that any attempt to teach a topic to uninterested pupils both puts a strain on the teacher and is without benefit to the learners.

Being interested in something is a feeling, an emotion. Our emotions are not at our beck and call. It is no use me saying, ***"I will tell you a story and you must try hard to be amused." Interest, like laughter and falling in love, is something that happens to us. Education results when adults are able to find the approach that will unlock the energy within a child and steer it into useful, or at least harmless, channels. This diversion of energy into acceptable channels is one of the most important aspects of teaching; it is a civilizing influence, and such influences have never been more needed than to-day.*** *(From Mathematics, Emotions, Things)*

Education is essentially the direction of mental energy. Children have abundant energy looking for an outlet…

At the various stages of development, a person's energies are concentrated on various objects which acquire the hue of romance – e.g., riding a bicycle, getting into a basketball team, love and courtship, …

Beauty is in the eye of the beholder. Any subject, any activity can acquire the halo of romance.

Thinking is extremely unsatisfactory and inefficient if the concentration of the mind by romance has not taken place.

(TPUML)

Therefore, the essence of good math teaching is 'motivation' and morale: *In a university lecture you can be sure all the appropriate results will be stated and proved. But the students are not always put in a position to see what the whole course is trying to do, where it came from and where it is going. I remember when I was at Cambridge I heard of only two lecturers who discussed the history of the subject.*

I would like to illustrate this point by discussing analysis. I believe that there was a survey which showed that the part of mathematics pupils enjoyed the most at school was calculus and at university what students enjoyed least was analysis. In school calculus is intuitive; you accept something if it sounds reasonable and it looks right. In universities it is exactly the opposite; everything must be proved with the utmost legalistic precision.

The schools approach corresponds to the way mathematicians worked in the 17th and 18th centuries; the universities to the way mathematicians thought in the 19th century.

Now in fact there is a very interesting explanation why, at a certain stage of history, mathematics changed from the first approach to the second. It arose from the interaction of music and mathematics. (From- Talk That Was Not Given)

Sawyer then goes on to narrate the origins of Fourier analysis and why and how this necessitates different concepts of convergence of functions. Everything he writes is in plain

language, delightful prose, behind which is a solid and deep understanding of mathematics.

Born in 1911, WW Sawyer graduated from Cambridge University with specialization in the applied mathematics of quantum mechanics and relativity. Immediately thereafter, he began his long career dedicated to teaching and learning math. Initially, he taught for several years in Britain. His first book *Mathematician's Delight* (1943) (MD), was written with the aim "to dispel the fear of mathematics."

Sawyer's second book, edited by him, and with six out of ten chapters written by him, *Mathematics in Theory and Practice* (1948) deals with the importance of making and constructing with materials for introducing concepts of school mathematics.

His third book, *Prelude to Mathematics* (1955) conceived while he was Head of the Department of Mathematics at the University College (Gold Coast, now Ghana) (1948-50) is about 'How to grow mathematicians.'

On a personal note: I was introduced to Sawyer's *Prelude* by an older friend and college mentor Nitant (now Prof. V.M. Kenkre, Emeritus Professor of Physics at UNM, USA) during my first year in college. The book had a profound impact – I felt exhilarated and elated after reading it, though I did not understand all of it at that time. (Many other writers who have written about this book describe experiencing the same feeling of elation after reading it.) Professors at Princeton, after reading *Prelude*, invited Sawyer to the USA to work on curriculum. Sawyer, at the time was at Canterbury College, New Zealand (1951-56).

Sawyer's writings cover a broad spectrum from primary math to 'advanced' math. In *Vision in Elementary Mathematics*, he shows how a simple game "Think of a number," can be translated into an introduction to algebra – how unknown numbers can also be represented by things, and can be added and subtracted much like pebbles. This writer has used this approach to introduce algebra to hundreds of primary school teachers. The universal response is "I never realized algebra is this simple."

Mathematician's Delight covers school geometry, including an introduction to Calculus. Mathematics at the secondary level is also covered fairly thoroughly in three other of his books: *Designing and Making* (1957, co-authored with Srawley), *What is Calculus About (*1961) and *The Search for Pattern* (1970).

College and University level mathematics is covered in four other books *A Concrete Approach to Abstract Algebra* (1959), *A Path to Modern Mathematics* (APMM) (1966), *An Engineering Approach to Linear Algebra* (1972) and *A Numerical Approach to Functional Analysis* (1978).

Each of Sawyer's books is a perennial classic. Before getting into the details and procedures of any topic in mathematics he gives the reader an idea of 'where it came from and where it is going.' There is one sentence that describes everything that Sawyer wrote – he was dedicated to conveying to the reader 'What is this all about' in the simplest possible way.

High priests of the subject may find his style infuriating – for example, in his chapter on Affine spaces (*The Arithmetic of Space*, APMM), he introduces vector addition by adding and subtracting cats and dogs, later going on to fractional and negative cats and dogs – but the ordinary reader will find herself exclaiming, "Oh, I never realized it was so simple!"

A few more examples from his writing will illustrate the above point.

The best way to learn geometry is to follow the road which the human race originally followed: Do things. make things, notice things, arrange things, and only then reason about things. (Mathematician's Delight. MD)

At the secondary level, Sawyer emphasised elementary approaches to calculus, again based on work with 12 to 15-year-old children. He

starts the discussion in ***MD*** with – *"The Basic Problem": The basic problem of differential calculus is the following: we are given a rule for finding where an object is at any time, and are asked to find out how fast it is moving.*

He summarises a beautiful discussion on complex function theory in the following words

There is a blanket theorem, which I have never seen stated in quite this way in any textbook:

Within a circle centred at the origin, not containing any singularity, you can safely carry out any operation on the power series that might occur to a sane mathematics student.
('The importance of the unbelievable')

This writer and the team at Navnirmiti were in touch with Prof. Sawyer through his daughter and son-in law during the final five years of his life. After Sawyer's demise, we were greatly honoured to receive a collection of his books, translations and handwritten notes, which are archived at the Sawyer Memorial in Pune. The articles referred to here are available at two websites www.marco-learningsystems.com, set up by Marc Alder, and www.wwsawyer.org, set up by Navnirmiti. VEM has been translated into Marathi. It is not easy to translate Sawyer. However, it will greatly benefit math education in India, if we can collectively translate his writings into Indian languages as soon as possible.

VIVEK MONTEIRO completed his doctorate in theoretical physics from the State University of New York, Stony Brook, in 1974. He returned to India and worked for two years at TIFR before switching fields to take up full time trade union work in 1977. He is currently Secretary, CITU Maharashtra State Committee. Through Navnirmiti, a self-reliant organization, of which he is a founder-advisor, he has continued working actively in math and science education. He is a trustee of the Sawyer Memorial trust.