Addendum to Math from Simple Grids

Exploring Addition - RowColSum Puzzle

ASHWIN, SHRAVAN, SHIVKUMAR & GOWRI SATYA

This is a quick puzzle that could be used at the end of a session. Again with a 3 x 3 grid and the numbers 1 to 9, the teacher sets a focus number and students race to find an arrangement of 1 to 9 in the grid such that the sum of the numbers in the same row and column as the focus number is the highest.

For example, in the grid in Fig. 19, if the focus number is 5:

RowColSum (5) = 5 + 1 + 8 + 4 + 7 = 25

1. Is this the maximum possible RowColSum of 5?
2. Is there a strategy to arrange the numbers in the grid so that RowColSum of 5 is maximum?
3. Does the strategy change when you change the position of 5?
4. Is there a 'preferred' position of 5?
5. Does the strategy change when the focus number is changed?

5	1	8
4	3	6
7	9	2

Figure 19

I find that some students even in Grades 3 or 4 have not fully assimilated the nature of addition. While they can perfectly reproduce the addition algorithm, the insight that "adding a bigger number to a given number results in a bigger sum" is missing. This is a fun puzzle to drive that point home.

You can play RowColSum puzzles online at:
http://mathventure.in/games/rowcolsum.html

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IV. Exploring Addition - Fubuki Addition Puzzle

The Japanese puzzle culture is rich in grid-based puzzles like Sudoku, Kakuro, etc. Another popular grid puzzle with Japanese origins is the Fubuki addition puzzle that is particularly suitable for the primary / middle school classroom.

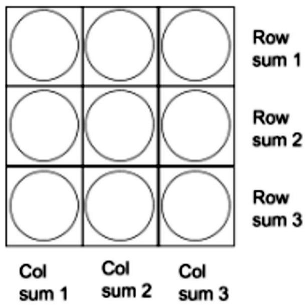


Figure 20

Given a 3 x 3 grid and target sums for each row and column, fill the grid with all the numbers from 1 to 9 such that the row and column sum targets are met. Each number can and should be used only once.

To start off, easy puzzles can be used where some of the numbers are filled into the grid (Fig. 21, Fig. 22) and slowly increased in complexity by decreasing the hint numbers in the grid (Fig. 23, Fig. 24).

Solving these puzzles requires students to make various connections and thrilling deductions.

1. What is the largest row or column sum possible? The smallest?

2. Can there be more than one solution?

3. Is there a pattern in the row and column sums?

One observation is that all four examples have two even numbers and one odd number in the row sums and similarly in the column sums. Does this always have to be so? Do the row sums always need to have the same pattern as the column sums in terms of parity? Of the 6 row / column sums, is it possible to have a configuration of say 3 odd numbers and 3 even numbers or 2 even numbers and 4 odd numbers?

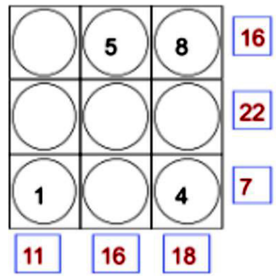


Figure 21

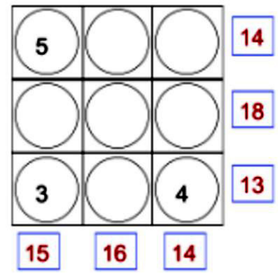


Figure 22

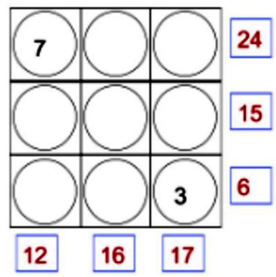


Figure 23

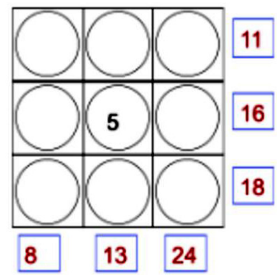


Figure 24

Again, we get to the most interesting part when we flip the question and ask students to create their own puzzles.

4. Can we have any randomly chosen six numbers as row and column sums?

Based on our previous observations, parity is one constraint. But what about the magnitude of the sums? Could there be a constraint on that? Observe the sum of the row sums and the sum of the column sums. Do they necessarily have to be that way? Why?

First, the sum of the row sums is equal to the sum of the column sums. This has to necessarily be so as it is the same nine numbers being added - just in a different order.

Second, the sum of the row sums and sum of the column sums have to necessarily be 45 as it is nothing but the sum of numbers 1 to 9.

5. Can we create a Fubuki addition puzzle that does not have any solution?

We have now identified two constraints on the row sums and column sums. Breaking either would lead to a puzzle without a solution.

6. How about if we remove the requirement of using numbers from 1 to 9?

If we could use any whole number inside the 3 x 3 grid, do the constraints on parity and magnitude of the row and column sums still hold? Can we still create an unsolvable puzzle under these revised conditions? How about if we expand the domain further - if we could have any integers inside the grid - how does this affect the solvability?

Challenge Question: *If any rational number could be used in the grid of a Fubuki addition puzzle, how does that affect the constraints on the row and column sums? What are the different ways to create a puzzle with no solutions?*

You can find Fubuki Addition puzzles online at: https://www.mathinenglish.com/puzzlesfubuki.php

V. Exploring Parity - Fubuki Addition Puzzle Variation

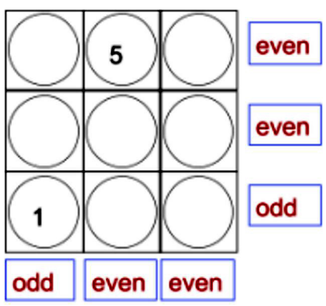


Figure 25

A more basic variation to the Fubuki addition puzzle - instead of specifying the row and column sums, we can just specify that they are to be even or odd. Fill the grid using the numbers from 1 to 9 exactly once such that the row and column sums match the parity specified. Again, similar questions pop up:

1. What are the constraints on the conditions?
2. Can we create a puzzle without a solution?
3. Can there be more than one solution to a puzzle?

Challenge Question: *If there can be more than one solution, how many solutions are possible? Is there an efficient way to find these?*

VI. Hollow Grid Puzzle

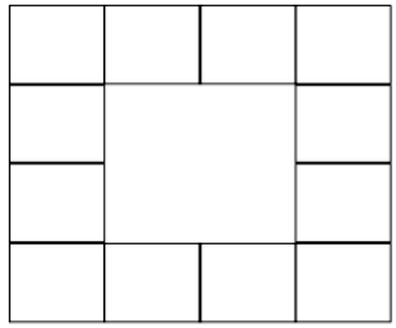


Figure 26

Given a 4x4 hollow grid (Fig. 26), fill each cell with pebbles such that each side of the grid has a total of 8 pebbles. Without actually solving the puzzle, can you predict how many pebbles are needed to do this?

Students are quick to use trial and error and come up with various solutions to this puzzle:

2	2	2	2
2			2
2			2
2	2	2	2

Figure 27

2	3	1	2
1			2
2			2
3	2	1	2

Figure 28

1	2	1	4
4			1
2			2
1	3	3	1

Figure 29

Now, if we add an extra pebble to an arbitrary cell in a given solution, is it possible to rearrange the pebbles so as to get the totals on each side back to 8? Amazingly, the extra pebble *can* (almost always) be accommodated. What if we add one more? This raises a host of questions:

1. **Can the required configuration be achieved for any and all numbers of pebbles?** Is there a minimum or maximum number of pebbles with which we can achieve this configuration? Can we achieve the 8 pebbles per side configuration for every number in between the min and max?
2. What's the key observation to help analyze and answer these questions?
3. For a given total number of pebbles, how do we come up with an arrangement that yields 8 pebbles per side? Is there more than one way to fill in the cells?

Let's take a look at the three solutions we have. What is the relation between the sum of the pebbles along each side and the total number of pebbles?

Fig. 27 - total number of pebbles = 24

Fig. 28 - total number of pebbles = 23

Fig. 29 - total number of pebbles = 25

Sum of pebbles along each side in every case is 4 x 8 = 32

It can be seen that the pebbles in the corner cells are counted twice and so, we have:

Total number of pebbles = sum of pebbles along each side - sum of pebbles in corner cells.

Using this, how many pebbles should be put in corner cells to accommodate the maximum total number of pebbles? How many in the corners for the minimum total number of pebbles?

4. What if we change one of the variables - each side needs to have 9 instead of 8 pebbles?
5. How about a bigger grid - say a 6x6 hollow grid (Fig. 30)? Can we still achieve a total of 8 pebbles along each side of the grid?

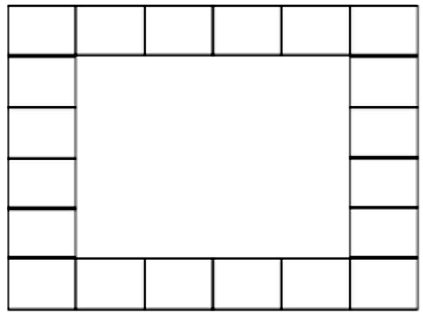


Figure 30

6. Is there a combination of grid size and side total for which there can be no solution? Can there be a grid with no solution if no empty cells are allowed?

I found that these puzzles could engage my students at many different levels. Each student learnt something new in the process and each student took home an unsolved question to mull on. The higher order questions about solvability have also kept me mulling for weeks. Now that makes for one happy math teacher.

Solutions

Some of these puzzles have multiple solutions but only one possible solution has been listed here.

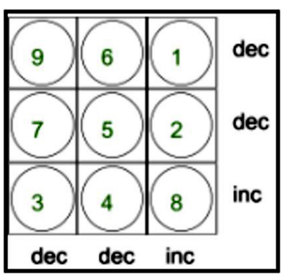


Figure 10

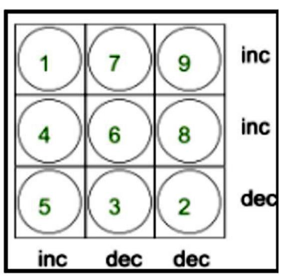


Figure 11

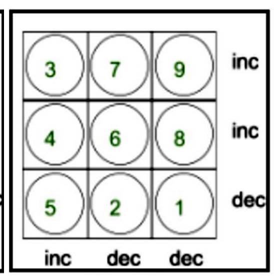


Figure 12

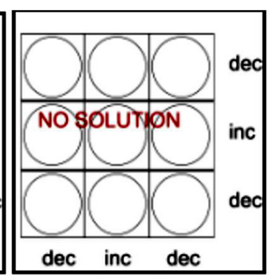


Figure 13

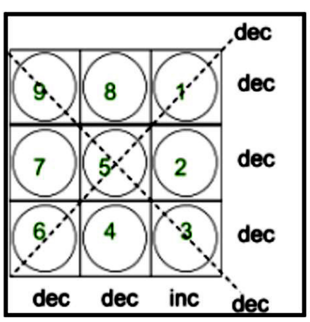


Figure 16

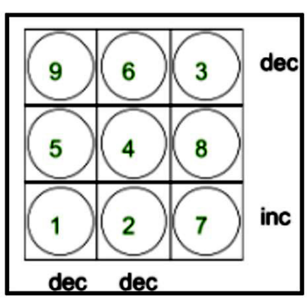


Figure 17

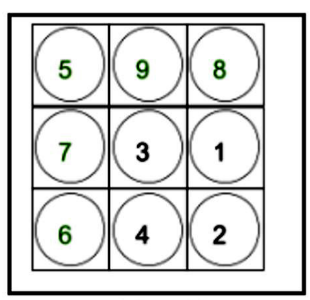
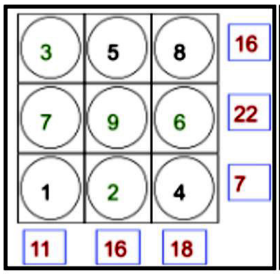


Figure 19



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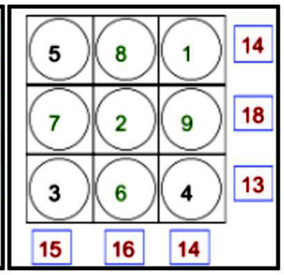

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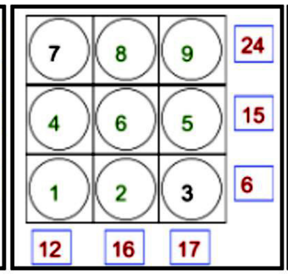

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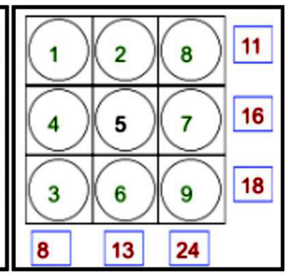
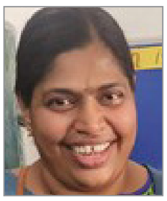


Figure 13

ASHWIN, SHRAVAN and **SHIVKUMAR** are all alumni of IISc, Bengaluru. They are now working full time on rejuvenating math education in schools through an education startup Seed2Sapling Education. They can be contacted respectively at Ashwin, guha.ashwin@gmail.com; Shravan, sra9449@gmail.com; and Shivkumar, shiv1729@gmail.com.



GOWRI SATYA teaches math at Swapaksh Learning Foundation, a learning center for underprivileged children in Bengaluru. An ex software engineer, she enjoys math and hopes to spread the same enthusiasm to her students. She can be contacted at s.gowri@gmail.com.