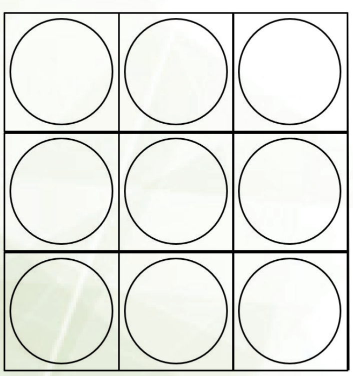
Maths from Simple Grids

GOWRI SATYA, ASHWIN, SHRAVAN, SHIVKUMAR

An exciting afternoon with friends and family, for me, often involves arguing endlessly over some trivial math or science question or solving a tricky puzzle or finding a pattern where you least expected it. When I started to teach math in an NPO (Non-Profit Organization) setting, I most of all wanted to bring this excitement and thrill of problem solving into our classroom. Easier said than done! I would pose what I thought was an interesting question and watch as students either struggled or simply did not even attempt it. Finding the right kind of problem that is challenging yet tantalizingly simple, that is familiar yet leads to new and exciting discoveries has proved to be difficult. Having to find something that is easy to implement, cost effective and that works for all the students in my mixed classroom even more so.

So, when Seed2Sapling Education conducted a teacher training session where they presented interesting, insightful math questions at various levels generated from common everyday things, I was completely hooked. Students' reactions to some of these ideas have been exactly what I had hoped for.



Here we present a few such interesting problems centered around the humble and modest **grid**. All the activities here can be worked out with paper and pencil. For younger or more kinesthetic learners, number tiles or even a life size grid on the classroom floor could be used. Answers to the puzzles are listed at the end of the article. Students from Grades 2 to 5 with a wide range of mathematical abilities can find something of appeal in these problems.

Keywords: critical thinking, problem solving, trial and error, generalisation

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

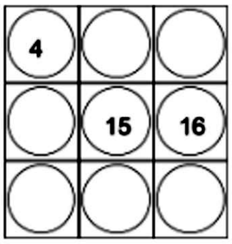


Figure 1

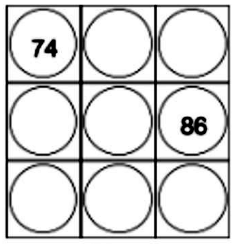


Figure 2

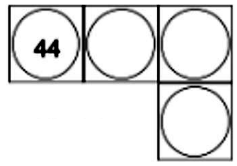


Figure 3

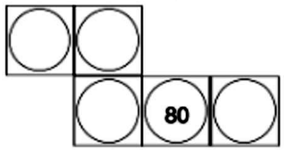


Figure 4

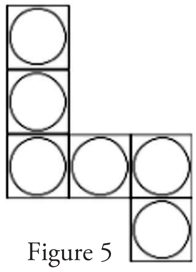


Figure 5

Preschool Corner - Number Grid Exploration

The most basic grid every child is bound to come across in school is the basic 1-100 number grid. While writing out the number chart is an oft repeated exercise, there are hours of fun to be had with finding patterns in it.

Given a 3 x 3 grid from the 10 x 10 number grid, fill in the missing numbers (Fig. 1, Fig. 2).

Or fill in the missing numbers in puzzle pieces of various shapes (Fig. 3, Fig. 4).

Or a puzzle piece can be left empty (Fig. 5). Then the challenge for the student is to find all the possible values that can work for it — first with a number grid for reference, then without.

Deeper understanding of the place value system can be fostered by asking students to explain in their own words their strategies for filling in the missing numbers. Mistakes made rather than just being corrected can be collected and students can observe for themselves the patterns of their mistakes. Again, asking students to explain in their own words what mistakes they made and why, can lead to rich math conversations.

Students can also design their own puzzle pieces. What is the most difficult puzzle piece you can come up with?

This same familiar activity can be repeated in later classes with an addition grid or a multiplication grid to bring out the patterns in addition and multiplication.

Exploring Ordering - Inc/Dec Puzzle

Given a 3 x 3 grid and the numbers 1 to 9 arrange them in the grid such that each row and each column has numbers in increasing order (Fig. 6). Each number can and should be used only once.

What if each row and column had to be in decreasing order (Fig. 7)? Now what if all rows were to be in increasing order and columns in decreasing order (Fig. 8, solved for reference)? Or vice versa (Fig. 9)?

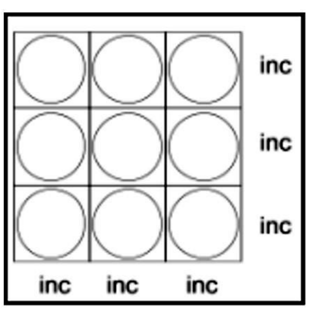


Figure 6

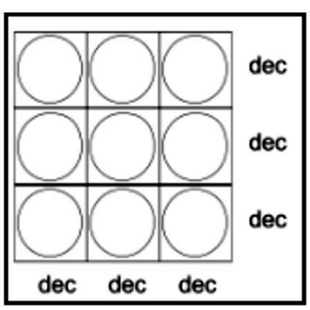


Figure 7

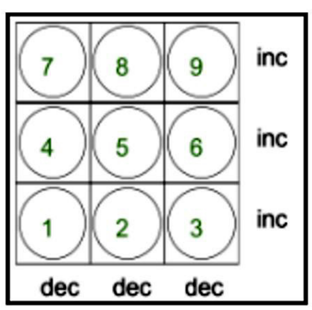


Figure 8

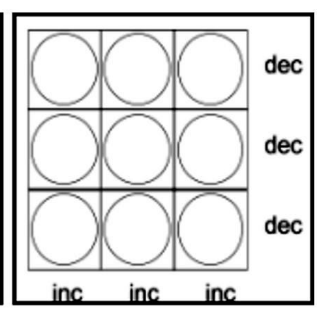


Figure 9

Now let us mix things up a bit. Have a go at the puzzles in Fig. 10 and Fig. 11. While the first four puzzles could be solved intuitively, the last two require a little more organized effort. A puzzle at this level would be ideal to introduce transitivity of ‘greater than’ and ‘less than’ to younger students.

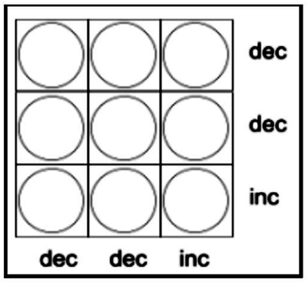


Figure 10

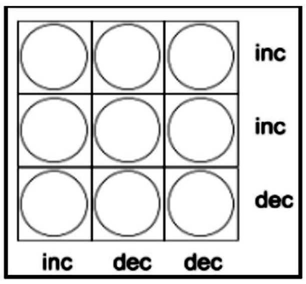


Figure 11

Now that we have six solutions under our belt, it is time to make some observations.

1. Are there any patterns in the solutions?

One thing that stands out is the placement of ‘1’ and ‘9’. Being the smallest and largest numbers in the set, they naturally take their positions in one of the four corners of the grid. Can the same be said of ‘2’ and ‘8’? Do they always take a position in the middle of a row or column?

2. Could a given grid configuration have more than one solution?

This is always an interesting question – can there be only one solution or are more solutions possible. Discussions around this can lead to insights that may be missed in just solving the problem.

3. Are there any patterns in our approach to solving them?

Based on our observations about 1 and 9, the first step could be to find possible positions for them. Next, try to find possible positions for 2 and 8 and so on. Or perhaps take the opposite route and shortlist the possible numbers for each cell in the grid. How about using transitivity of the ‘less than’ and ‘greater than’ to solve the puzzles?

Once students have solved a few puzzles and understood the constraints, what works, what does not, it’s time to up the ante. Let us flip the question. Create your own puzzle. Students can create a grid with the conditions of increasing and decreasing and give it to a friend or teacher to solve. (What student wouldn’t like to school her teacher?) Some interesting grids that students came up with:

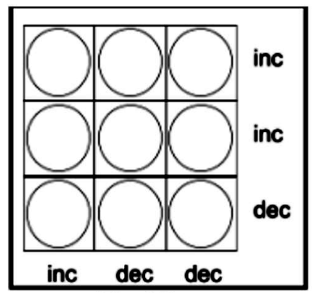


Figure 12

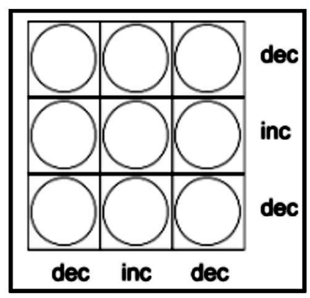


Figure 13

This exercise opens up some more questions:

4. How many ways are there to arrange Inc and Dec around a grid to set a puzzle like this?

There are 3 rows and 3 columns giving in all 6 positions to be filled with one of two values - ‘Inc’ or ‘Dec’. What would be the total number of combinations?

If there were 2 positions to fill, we would have 2 x 2 = 4 possible combinations:

Inc Inc, Inc Dec, Dec Inc, Dec Dec

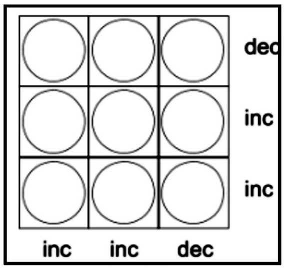


Figure 14

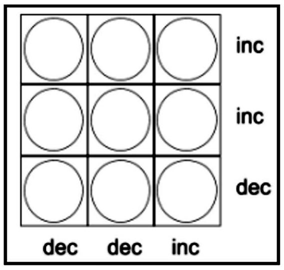


Figure 15

If there were 3 positions to fill, we would have an Inc- extension to all the above sequences and then a Dec- extension to all the above sequences like this:

Inc Inc Inc, Inc Inc Dec, Inc Dec Inc, Inc Dec Dec

Dec Inc Inc, Dec Inc Dec, Dec Dec Inc, Dec Dec Dec

That is, with each extra position that is added, the number of combinations doubles.

This gives us $2^6 = 64$ combinations for 6 positions.

5. Will all of those combinations have a solution or is it possible to create a puzzle with no solution?

Given that the two conditions of 'increasing' and 'decreasing' are contradictory, it seems likely that a particular configuration could lead to a contradiction that makes the puzzle unsolvable. Let's try to create one then. After some attempts, we hit upon the configurations shown in Figures 14 and 15. The proofs that these configurations are unsolvable is given separately at the end (see the Appendix, last page). The reader may enjoy looking for independent proofs.

6. How does adding or removing constraints affect the puzzle?

Along with the row and column constraints, we could add a constraint on the diagonals as well. Consider Fig. 16. Is this harder to solve? Can we create more unsolvable puzzles with the extra constraints?

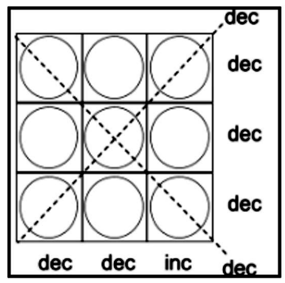


Figure 16

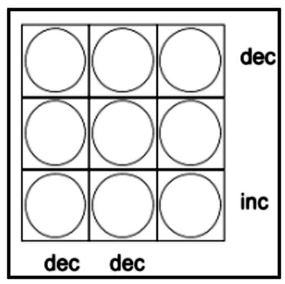


Figure 17

What if we drop a constraint? Instead of specifying Inc / Dec on every row and column, we could leave some with no constraints as in Fig. 17. Does this make the puzzle easier to solve? Is it harder to create an unsolvable puzzle if you could impose only 4 constraints instead of 6.

Challenge Question: How do we efficiently identify whether a given configuration can be solved or not?

We have one way to identify an unsolvable puzzle but is that the only condition resulting in no solutions? Consider the puzzle in Fig.18. Here, 9 and 1 have clearly identifiable cells that they could fill. Does that make this a solvable puzzle? Could there be a contradiction elsewhere?

How can we efficiently find out if a given configuration is solvable? Can this be extended to a generalized Inc/Dec puzzle on a $n \times n$ grid or even further to a $n \times m$ rectangular grid?

It turns out that this question has an unexpected twist!

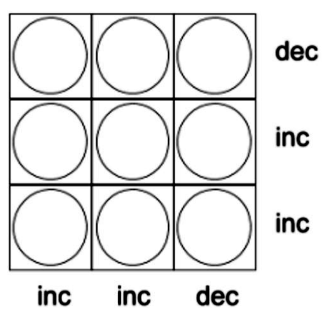


Figure 18

You can play Inc/Dec puzzles online at: http://mathventure.in/games/incdec.html

Solutions

Some of these puzzles have multiple solutions but only one possible solution has been listed here.

For more puzzles, please visit the addendum to this article in the online version available at http://publications.azimpremjifoundation.org/3344/

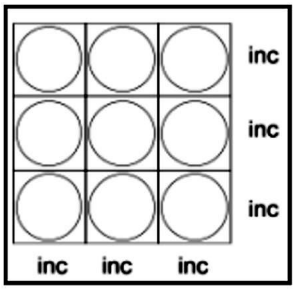
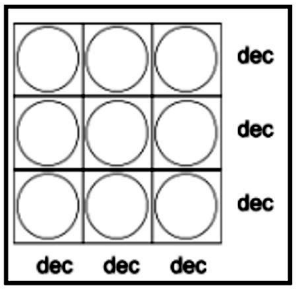


Figure 6



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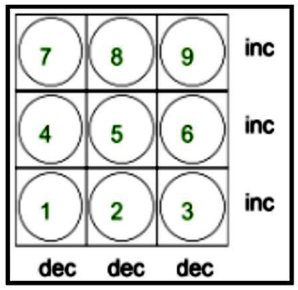

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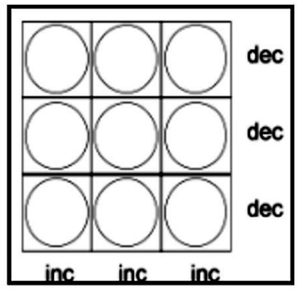

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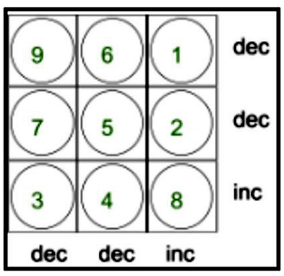

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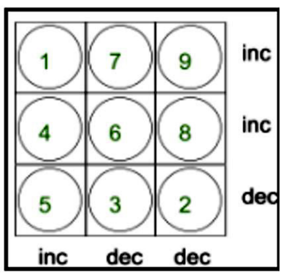

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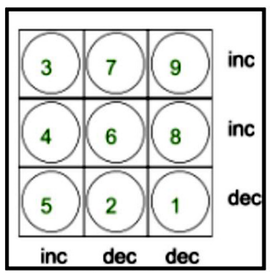


Figure 12

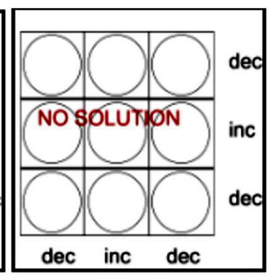


Figure 13

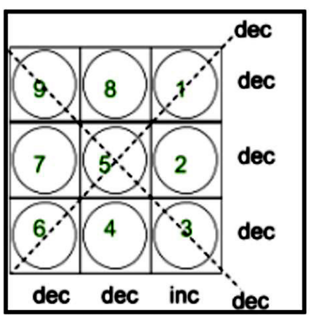


Figure 16

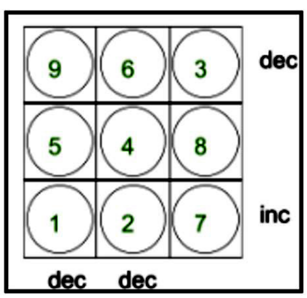


Figure 17

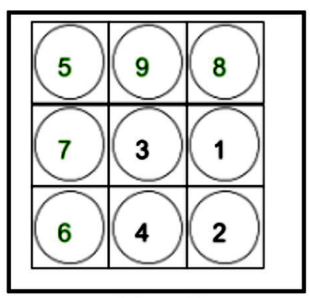


Figure 19

Appendix: Proof that the configurations shown in Figure 14 and Figure 15 are not solvable

Let the grid be as follows.

a	b	c	**Dec**
d	e	f	**Inc**
g	h	i	**Inc**
Inc	**Inc**	**Dec**	

Let's look at Figure 14 first (shown alongside). Since column #1 is Inc and row #1 is Dec and column #3 is Dec, it must be that:

$$i < f < c < b < a < d < g.$$

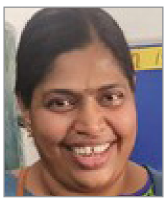
This implies that $i < g$. However, row #3 is Inc, which implies that $g < i$. These two conclusions are contradictory. Therefore, this configuration is not solvable.

Similarly, in Figure 15, since column #1 is Dec and row #1 is Inc and column #3 is Inc, it must be that:

$$i > f > c > b > a > d > g,$$

implying that $i > g$. However, row #3 is Dec, which implies that $g > i$. Once again, we reach a contradiction. Therefore, this configuration too is not solvable.

ASHWIN, SHRAVAN and **SHIVKUMAR** are all alumni of IISc, Bengaluru. They are now working full time on rejuvenating math education in schools through an education startup Seed2Sapling Education. They can be contacted respectively at Ashwin, guha.ashwin@gmail.com; Shravan, sra9449@gmail.com; and Shivkumar, shiv1729@gmail.com.



GOWRI SATYA teaches math at Swapaksh Learning Foundation, a learning center for underprivileged children in Bengaluru. An ex software engineer, she enjoys math and hopes to spread the same enthusiasm to her students. She can be contacted at s.gowri@gmail.com.