Problem Corner

Six Problems on Area and Perimeter

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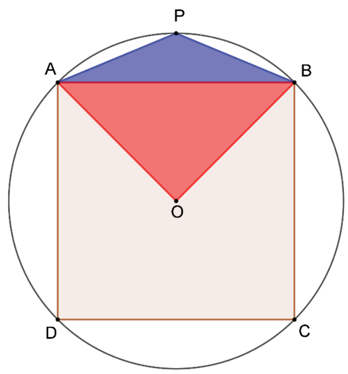
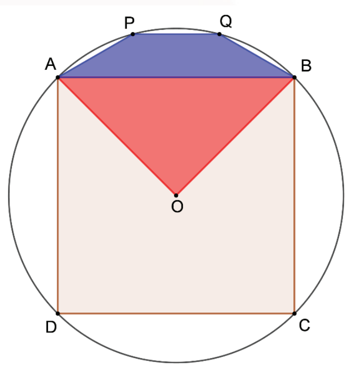
The following formula for the area of a triangle could be useful to you in solving the first two problems.

$$\text{Area of } \triangle ABC = \frac{1}{2}\text{ab sin C.}$$

1. The accompanying figure shows a square ABCD inscribed in a circle with centre O. P is the midpoint of minor arc AB. Find the ratio of the areas of △AOB and △APB.

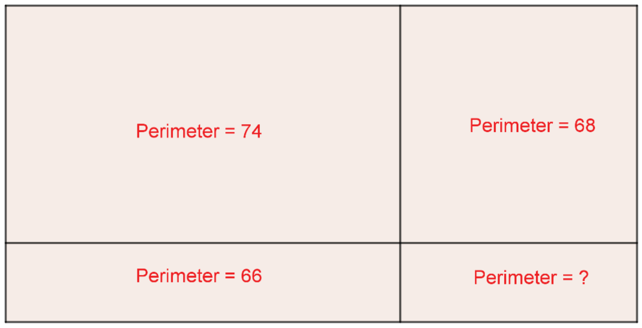


2. The accompanying figure shows a square ABCD inscribed in a circle with centre O. Points, P, Q are situated in the minor arc AB such that AP = PQ = QB. Find the ratio of the areas of △AOB and quadrilateral △APQB.



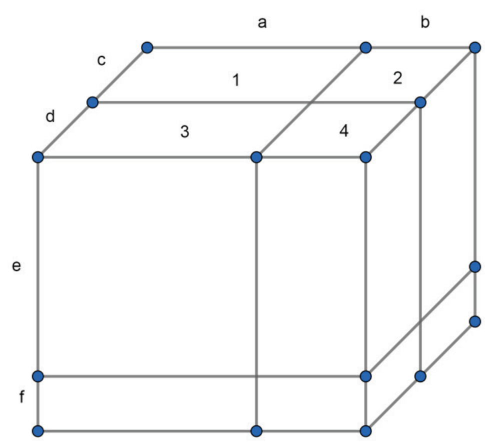
Keywords: Area; Perimeter; Triangle; Trigonometry; Ratio; Composition; Decomposition; Dissection

3. In the accompanying figure, a rectangle is divided into four smaller rectangles. If the perimeters of the top left, top right and bottom left rectangles are 74, 68 and 66 units respectively, find the perimeter of the rectangle at bottom right.



4. With reference to the figure of the previous question, if the areas of the top left, top right and bottom left rectangles are 135, 108 and 120 square units respectively, find the area of the rectangle at bottom right.

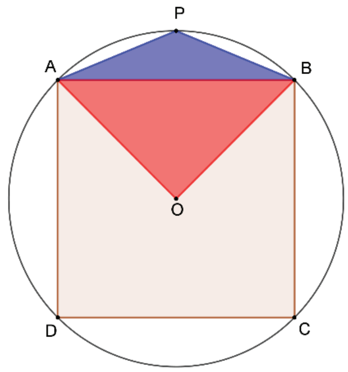
5. This problem is a 3-D analogue of the previous problem. We have a cuboid divided into eight smaller cuboids by three plane cuts one parallel to each pair of opposite faces. (See Figure.) The eight smaller cuboids have been numbered from 1 to 8. Cuboids 5,6,7,8 are directly below cuboids 1,2,3,4 respectively. The different edge lengths have been designated as a,b,c,d,e,f. If now the volumes of cuboids 1, 2 and 7 are given, how can you find the volume of cuboid 8?



6. This problem relates to the dissected cuboid of the previous problem. If the total surface areas of cuboids 1 to 7 are given, how can you calculate the total surface area of cuboid 8?

Solutions

1. Let us take the radius of the circle to be 1 unit.



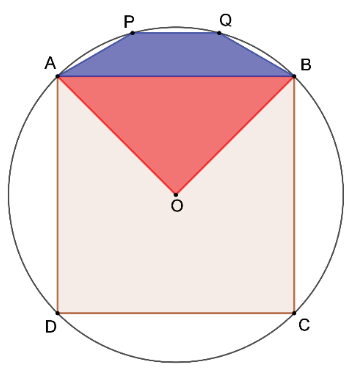
Then area of $\triangle$ AOB $= \frac{1}{2}$ square units. If we join OP, $\measuredangle$ AOP $= 45°$, by symmetry. Then area of $\triangle$ AOP $= \frac{1}{2}(1)(1)\sin 45° = \frac{1}{2\sqrt{2}}$.

Area of quadrilateral APBO $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

Area of $\triangle$ APB $= \frac{\sqrt{2}}{2} - \frac{1}{2} = (\sqrt{2} - 1)/2$. [We could also find the length of the altitude on base AB in $\triangle$ APB and thereby the area of $\triangle$ APB.]

Ratio of areas of $\triangle$ AOB and $\triangle$ APB $= 1 : \sqrt{2} - 1$.

2. We use an approach similar to that of the earlier solution. If we join OP, then $\measuredangle$ AOP $= 30°$ (equal chords subtend equal angles at the centre).



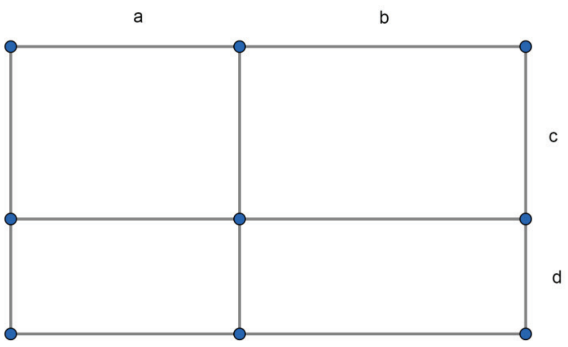
Area of $\triangle$ AOP $= \frac{1}{2}(1)(1)\sin 30° = \frac{1}{4}$. (Radius of circle taken as unity.)

Area of pentagon APQBO is then $\frac{3}{4}$.

Area of quadrilateral APQB $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$. Ratio of areas of $\triangle$ AOB and quadrilateral APQB $=$ 2 : 1.

3. Let us name the different lengths in the figure as a, b, c and d. (See accompanying figure.) Then we have $2(a+c) = 74$; $2(b + c) = 68$; $2(a + d) = 66$.

The required result is $2(b + d) = 2(b + c) + 2(a + d) - 2(a + c) = 68 + 66 - 74 = 60$.



4. Using the same designation of lengths as in the previous solution, we have $ac = 135$; $bc = 108$; $ad = 120$. The required result is $bd = bcad/ac = (108 \times 120)/135 = 96$ square units.

5. Volumes of cuboids 1, 2 and 7 are ace, bce and adf respectively. Volume of cuboid 8 is bdf. Notice that

$$\frac{bce \times adf}{ace} = bdf. \text{ That is, } (\text{volume}_2 \times \text{volume}_7) \div \text{volume}_1 = \text{volume}_8.$$

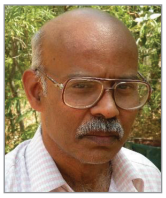
In other words the product of the volumes of two cuboids that are diagonally opposite to each other (do not share a face or edge but meet at a vertex) is the product of all six edge lengths, i.e., $abcdef$.

6. Let us write down the TSA (total surface area) of all eight cuboids, using the length designations of the previous problem:

$$\begin{aligned} \text{TSA}_1 &= 2(ac + ae + ce); & \text{TSA}_2 &= 2(bc + be + ce); \\ \text{TSA}_3 &= 2(ad + ae + de); & \text{TSA}_4 &= 2(bd + be + de); \\ \text{TSA}_5 &= 2(ac + cf + af); & \text{TSA}_6 &= 2(bc + bf + cf); \\ \text{TSA}_7 &= 2(ad + af + df); & \text{TSA}_8 &= 2(bd + bf + df). \end{aligned}$$

Note that $\text{TSA}_1 + \text{TSA}_4 + \text{TSA}_6 + \text{TSA}_7 = \text{TSA}_2 + \text{TSA}_3 + \text{TSA}_5 + \text{TSA}_8$.

So, knowing any seven TSA's, one can obtain the remaining TSA.



A. RAMACHANDRAN has had a longstanding interest in the teaching of mathematics and science. He studied physical science and mathematics at the undergraduate level and shifted to life science at the postgraduate level. He taught science, mathematics and geography to middle school students at Rishi Valley School for two decades. His other interests include the English language and Indian music. He may be contacted at archandran.53@gmail.com.