

The Educational Use of Tables on Advanced Scientific Calculators

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Although advanced scientific calculators of recent years include many features intended for educational use, they are still frequently misunderstood as mostly devices for numerical computation. Following a model for the educational use of calculators, this article describes and illustrates several ways in which the use of a calculator facility to construct tables of values can have educational value in the secondary school. Examples include the study of linear and quadratic functions, algebraic equivalence, equations, sequences, series, limits, convergence, differentiation and integration. The article concludes by observing that the educational value of calculators derives from the experiences they offer students, not merely from their capacity to generate numerical answers.

Introduction

Hand-held calculators have been around for almost fifty years now. It was no surprise that they quickly replaced older technologies for computation when they first appeared in the 1970s, as they were much easier to use and much more accurate than alternatives. However, it is a surprise that the calculators continue to be described as *scientific* calculators, when their many significant developments and improvements since then have been concerned almost entirely with *education* and not with science. The most obvious improvements to scientific calculators over the last fifty years include increased functionality (what a calculator can do), increased usability (how easy it is to do it) and the use of natural displays (so that calculator screens increasingly show mathematics as it is represented elsewhere).

There have been significant expansions of scientific calculators to include a graphics screen and even to include computer algebra capabilities, but we restrict attention in this paper to devices that are still described as scientific calculators. In particular, we focus attention on one of these developments, the capacity to display tables of numerical values, and elaborate on some of the ways in which this capability is of value to school students of mathematics.

Keywords: calculators, pedagogy, functions, tables, variables

Calculators and education

Naïve thinking about calculators still regards them solely as devices for arithmetical calculation. In contrast, Kissane & Kemp (2014) elaborated a model for the educational role of calculators, highlighting three other aspects of calculators that are also important. Computation is helpful for students, especially when the numbers involved are awkward for mental or by-hand calculation, such as when real measurements are involved or students are dealing with genuine statistical data, rather than fabricated data of the kinds that often appear in textbooks. The other three aspects highlighted by the model are Representation, Exploration and Affirmation. Calculators allow mathematical concepts to be represented in various ways, helpful to learners; a good example is the use of a table of ordered pairs to represent a function, in addition to symbolic representations. Exploration of mathematical relationships and ideas has long been recognised as a powerful means of active learning and is at the heart of many student-centred approaches to learning. As this paper illustrates, tables offer many opportunities of this kind. Finally, calculators can be used thoughtfully by students engaged in affirmation of their own thinking, which also involves the possibility of contradiction of their own thinking; the responsiveness of calculators allows tables to be used to permit students to test out their own thinking and to engage actively in their own learning.

Advanced scientific calculators

Calculators are routinely improved by manufacturers, with advice from teachers and mathematics educators, so that modern versions are at least a fifth generation of the species. In recent years, some manufacturers have developed *advanced* scientific calculators with suites of mathematical capabilities focused on senior secondary school or the early undergraduate years. These have been mostly developed for environments in which graphics calculators are prohibited for use in formal examination systems. Student use of such calculators might

be preferred over more sophisticated forms of technology in some settings: they are much less expensive than technology requiring individual computer access; they are more acceptable to some examination boards than are computers; they are sufficiently portable to be easily taken by students between classroom, home and examination room.

Many advanced scientific calculator models (and some less advanced models) from different manufacturers now include a capacity for users to generate tables of values of functions, which is the main focus of this paper, and to which attention now turns.

Using tables

It is now almost twenty years since the first scientific calculators included a facility to construct tables of values, originally described by the manufacturer (CASIO) as ‘number tables’. Mechanisms for defining tables differ between calculator manufacturers, with some tables defined with a finite domain, while others are defined with a ‘scrolling’ infinite table. Regardless of this, typically, the small screen size of calculators constrains tables to showing only a few terms at any time, so that scrolling to see further rows of a table is generally necessary. The essential user input to generate tables is a definition of a function and a mechanism to determine which values will be tabulated.

Not all calculators are the same, as noted above, so for convenience in this paper we will use a particular calculator, the CASIO fx-991 EX *ClassWiz*, as a device to illustrate the key ideas. While relatively modern, this advanced scientific calculator has now been in regular educational use in many countries for at least seven years and is described in some detail in Kissane (2015). Although the first scientific calculators that included a capacity to generate tables were restricted to a single table, while graphics calculators generally permit three or more tables, *ClassWiz* allows users to generate two tables simultaneously. It should be noted that some scientific calculators intended for younger

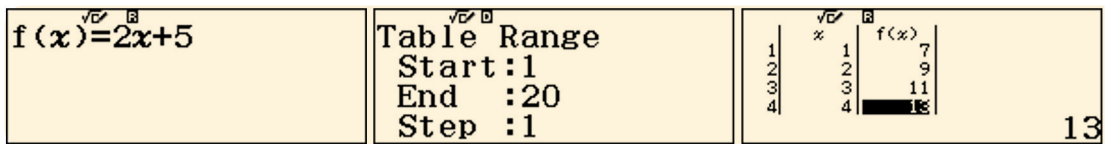


Figure 1: Defining a table on *ClassWiz*

students also include a capability to generate tables, so that some parts of this paper are also relevant to the earlier years of secondary school. In the remainder of this paper, brief glimpses of the potential educational benefits of a table capability are described and illustrated.

Understanding linear functions

Perhaps the most obvious use of tabulation is to explore the nature of functions. Essentially, a table of values for a function allows a user to generate a set of ordered pairs for the function, according to a defining rule. On *ClassWiz*, the process is quite direct, as shown in Figure 1, with a linear function (of x) defined on a finite domain generating a table of values instantly.

Early discussions about the use of computers and graphics calculators to represent and study functions frequently referred to a “Rule of Three”, referring to symbolic, numerical and graphical representations. The representation of a function in this way on the calculator helps users to regard it differently from the symbolic representation, and of course provides an efficient way for users to construct a suitable graphical representation, avoiding the need for extensive by-hand computation.

The educational value of the table extends beyond its efficiency for drawing a graph, however. In this case, the numerical values displayed allow users to see for themselves that the function values $f(x)$ increase by 2 when the value of the independent variable, x , increases by 1... which is the essential idea of the *slope* of the function, as well as the key idea that the increase is steady, and does not vary with changing x .

While the screen limitation restricts *ClassWiz* to displaying only four ordered pairs, this affords users an opportunity to test their understanding

of the function, by predicting the next term, in this case the value associated with $x = 5$ before using the cursor key to scroll down. In a similar way, *ClassWiz* allows users to change an x -value in a table directly (by entering a new value directly) in order to see its associated $f(x)$ value. Both of these operations are shown in Figure 2.

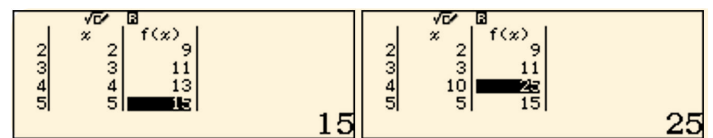


Figure 2: Exploring a table

It seems likely that most users would predict the next term of the table in the first screen, by continuing the process of increasing the previous value by 2 to predict that $f(5) = 13 + 2 = 15$. It also seems likely that users might predict $f(10) = 2 \times 10 + 5 = 25$. Both of these illustrate the idea of affirmation referred to in the model and make clear that the calculator allows students to manipulate what they see and to learn from it. Of course, the environment allows students to easily explore other linear functions, such as $f(x) = 2x + 6$ or $f(x) = 3x + 5$ in order to see for themselves how these differ from each other.

While such explorations are possible when a calculator has a tabulation facility, they are enhanced when a calculator, such as *ClassWiz*, allows for a pair of simultaneous tables to be generated, as illustrated in Figure 3, with $f(x) = 2x + 5$ and $g(x) = 2x + 6$.

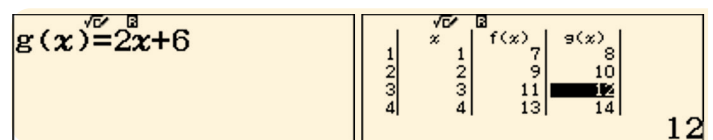


Figure 3: A pair of tables

Users can see for themselves in Figure 3 that the function values for g are always one more than

those for f , so that $g(x) = f(x) + 1$. They can also see that each function behaves in a similar way, which would be reflected in identical slopes of 2 when they are graphed. These observations can be readily tested – and affirmed – as before. Such an environment offers powerful and readily accessible learning insights for students on the nature of linear functions and the significance of their parameters.

Quadratic functions

Opportunities provided by a calculator for linear functions are of course also available for other species of functions typically studied in the secondary school years. Essentially all school curricula proceed from linear functions to quadratic functions, before examining other kinds of functions. The contrast between linear and quadratic functions is valuable for students, as these are used to model quite different kinds of phenomena and are represented by quite different kinds of graphs. To illustrate, the screens in Figure 4 show a table of values for $f(x) = x^2 + 3x + 2$ for $1 \leq x \leq 4$.

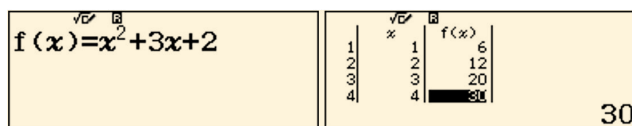


Figure 4: Tabulating a quadratic function

The table makes it clear that, unlike linear functions, the values of f do not change in the same way as x increases, helping users to understand why the associated graph would not be linear. The values do, however, show a consistent pattern, with the differences between successive function values changing in a systematic way. Users can see the pattern of increases, +6, +8 and +10 in the few table values shown in Figure 4. They might then predict that later increases will change in the same way, expecting +12, +14, +16, etc. To affirm that these predictions are correct involves merely scrolling down the next few terms, as shown in Figure 5. Collectively, these data give a strong suggestion that the function is increasing in an increasing way, helping to anticipate the shape of the graph.

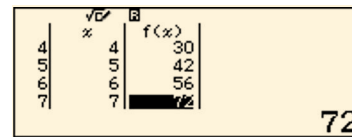


Figure 5: Extending a table to affirm predictions

Further evidence of the quadratic behaviour, and its differences from linear behaviour, can be found by students exploring different values of the independent variable, x , as shown below, with $-6 \leq x \leq -3$. Here the table shows that successive values are decreasing, instead of increasing, although still changing in what seems to be a predictable way, with changes of -8 , -6 and -4 shown in the first screen in Figure 6.

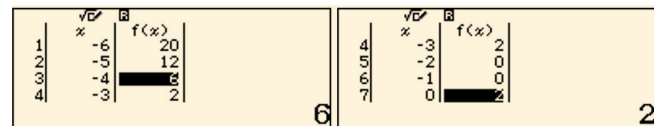


Figure 6: Further explorations of a quadratic function

When further tabulated values are revealed (by scrolling), as shown in the second screen shot in Figure 6, further phenomena are revealed, at first unlikely to be expected by naïve users. In this case, two function values are the same, $f(-2) = f(-1) = 0$. Furthermore, successive function values stop decreasing and start increasing and there is a clear symmetry of values shown.

These kinds of explorations provide students with a chance to understand some features of this particular quadratic function, and of its graphical representation, such as its general shape as well as whether and where it crosses the x -axis. Users can easily conduct similar explorations with different quadratic functions to gain even further insights into this family of functions. Explorations of these kinds are much more difficult if a tables facility is not available, requiring extensive by-hand computations.

Trigonometric functions

As for algebraic functions, the availability of tables allows a calculator to be used to get insight into the values of trigonometric functions. Although some insight is available via a conventional book of tables, a calculator allows

for a larger range of values to be explored. An example is shown in Figure 7, where the sine function has been tabulated for $0^\circ \leq x \leq 435^\circ$. The *ClassWiz* is constrained to a table of no more than 30 entries, so a suitable choice of increment is shown.

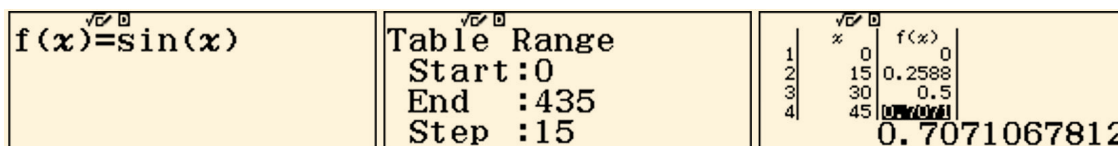


Figure 7: Tabulating the sine function

Scrolling a table like this allows a user to see that sine values increase from 0 at 0° to a maximum of 1 at 90° , and then decrease in the same way to 0 again at 180° . Sine values in the third and fourth quadrants are readily seen to be the opposite of those in the first two quadrants – and are always negative. The periodic nature of the function can be glimpsed by examining further values for $x > 360$. Of course, a table like this is not only helpful for visualising the sine function, but also for sketching it on paper. Finer-grained increments can be used if desired to obtain more precision.

Using a pair of tables of trigonometric functions offers users a chance to see some relationships between the functions, as the tables in Figure 8 of both the sine and cosine functions for $0^\circ \leq x \leq 180^\circ$ illustrate.

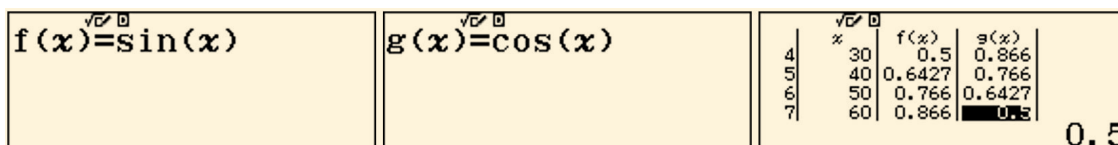


Figure 8: Tabulating the sine and cosine functions together

Exploring tables like these allow users to see that values for sines and cosines repeat; closer analysis will suggest further detailed relationships such as $\sin x = \cos(90^\circ - x)$, worthy of exploration in other ways.

Equivalence

As well as representing and exploring functions of various kinds, calculator tables offer an opportunity for the key notion of equivalence to be represented and understood. Researchers have long recognised the limitations of students regarding algebra as essentially a process of operating on symbols, or letters, but with little sense of the meaning of the operations used. However, a numerical meaning of equivalence can be provided by evaluating algebraic expressions at many points. An example is shown in Figure 9, in which two functions are tabulated.

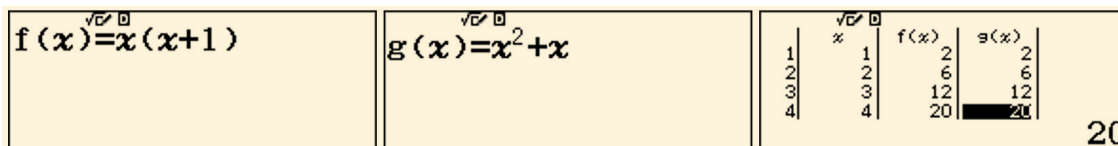


Figure 9: Tabulating a pair of equivalent functions

Inspecting the tabulated values, and scrolling to see values for other points, allows users to see that they are always the same for the two functions. This provides a powerful stimulus to understanding the meaning of an identity such as $x(x+1) = x^2 + x$; regardless of the value of x , the two functions have the same value, helping students to see that processes such as ‘expanding’ and ‘factorising’ algebraic expressions are merely ways to represent them differently. The screens in Figure 10 show a similar process for the quadratic function described in the previous section:

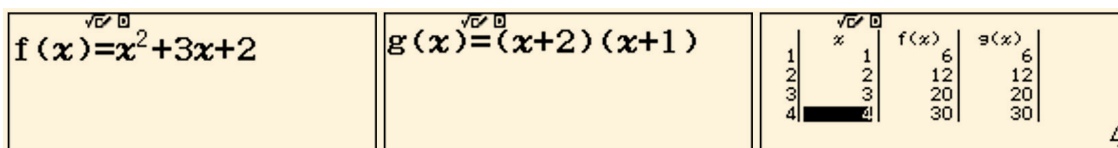


Figure 10: Exploring factorisation through tables

In a comparable way, users can explore for themselves some algebraic operations, to have their processes confirmed, or, as in the case shown in Figure 11, contradicted.

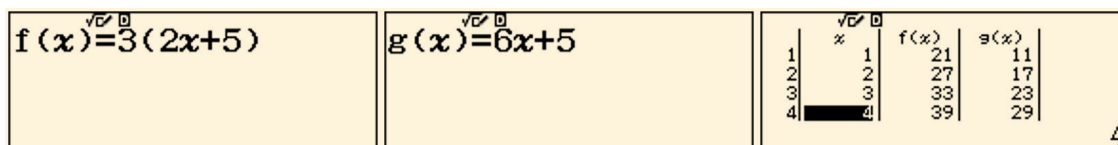


Figure 11: Checking equivalence through tables

As any teacher will attest, many students have difficulty with expressions like $3(2x + 5)$, because of conceptual errors, suggesting that an equivalent expression is $6x + 5$. Clearly, the calculator offers significant evidence that this is not the case. Using tables on calculators to explore these – and many other – examples offers another way of thinking about the matter. Of course, there is no suggestion here that students do not need to learn the symbolic manipulative processes eventually; rather, it is suggested that the calculator will help them to make sense of what the equivalence processes actually mean.

The same idea might be used to consider trigonometric identities too. The example in Figure 12 shows how students can easily see that, for almost all values of x , $\sin 2x \neq 2\sin x$.

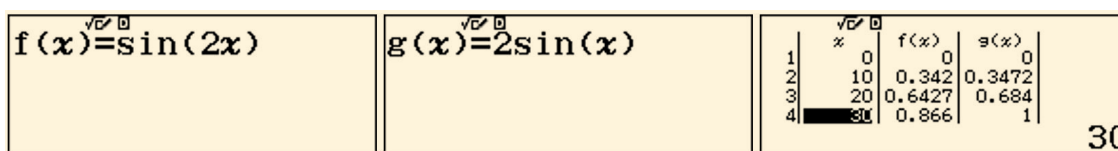


Figure 12: Exploring a trigonometric relationship

Identities can be affirmed as well, of course. The example in Figure 13 illustrates the Pythagorean relationship that $\sin^2 x + \cos^2 x = 1$. (Note that the conventional – albeit idiosyncratic – syntax of $\sin^2 x$ does not represent $\sin(\sin(x))$, but rather means $(\sin x)^2$. On *ClassWiz*, the corresponding Pythagorean expression is written in calculator syntax as $\sin(x)^2 + \cos(x)^2$, shown over the first two screens in Figure 13.)

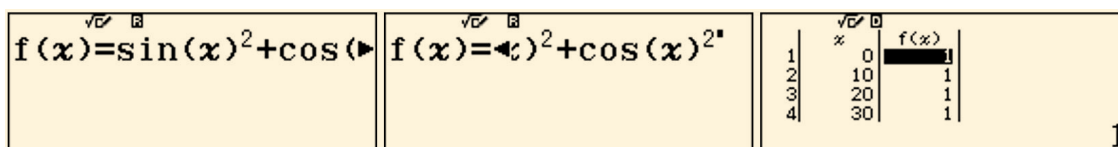


Figure 13: Using tables to represent the Pythagorean identity

Again, the calculator is supporting learning about these relationships; in other parts of student learning experience, alternative approaches will be used as well, including formal proofs. The fundamental role of the calculator is to help the user make sense of the mathematical ideas and relationships involved.

Solving equations

Solving equations is a key component of any secondary school mathematics curriculum, but attention is rarely focused on the very limited range of equations that is generally included. Table capabilities on calculators add a further dimension to equation solving, allowing numerical approximations to essentially any elementary equation that makes sense to students.

Consider for example the equation $x^3 = 3 - x^2$, which is generally beyond the capabilities of typical secondary school students to solve. Solutions, if any, to the equation will be the (irrational) roots of the function $f(x) = x^3 + x^2 - 3$. As shown in Figure 14, scrolling a table of values for this function on a wide interval, $-10 \leq x \leq 10$, suggests that there is a root between $x = 1$ and $x = 2$, as the function changes sign in this interval:

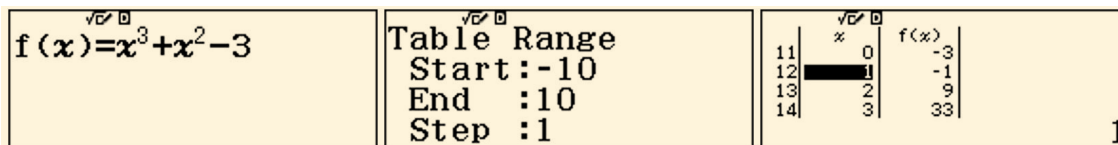


Figure 14: Using a table to find approximate roots of a function

To obtain a closer approximation, users can change the table interval to $1 \leq x \leq 2$, with a smaller increment of 0.1 and redraw the table, as shown in Figure 15.

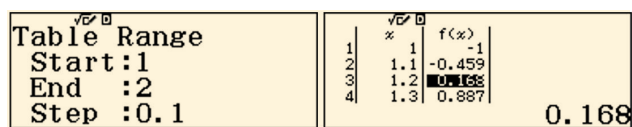


Figure 15: Refining a table to improve precision

Now it is clear that a solution (root) lies between $x = 1.1$ and $x = 1.2$. Redrawing the table again with the interval $1.1 \leq x \leq 1.2$ and a smaller increment of 0.01 allows a better approximation of $1.17 < x < 1.18$ to be made. A further refinement is shown in Figure 16, to demonstrate that there is a root in the interval $1.174 < x < 1.175$.

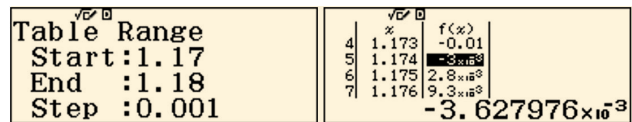


Figure 16: Zooming in to show that a root lies in $1.174 < x < 1.175$

This process of ‘zooming in’ on the table can be repeated efficiently, and can be used to improve the approximation to a solution by one decimal place each time, to whatever accuracy is regarded as appropriate. Using this process, a good approximation to an irrational solution of the equation is found at $x \approx 1.1746$, which is correct to four decimal places.

Although some calculators (including *ClassWiz*) include numerical equation solving software, using a table of values in this way provides the user with control of the approximation process and a solid understanding of the nature of a solution, which are less likely to characterise other solution methods.

Sequences and series

Many curricula include the study of sequences and series, although typically they are restricted to arithmetic and geometric sequences. While students are expected to use formulas of various kinds to answer questions of interest, using tables on calculators provides an alternative – and frequently more insightful – approach. Essentially, if students know how to find a general term of a sequence, as a function of its place in the sequence, then the calculator can be used to generate terms explicitly and answer many questions of interest by inspecting the table.

To illustrate, consider a geometric progression with first term 3 and common ratio 2, so that each term is twice the previous term. Students are sometimes asked to find a particular term (such as the 12th term) or terms with certain properties (such as the first term larger than six lakhs). The n th term of the sequence is $T_n = 3 \times 2^{n-1}$, $n = 1, 2, 3, \dots$. To generate successive terms of this sequence on the calculator, a function of x is needed, as shown in Figure 17.

$f(x) = 3 \times 2^{x-1}$	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td>10</td><td>1536</td></tr> <tr><td>11</td><td>3072</td></tr> <tr><td>12</td><td>6144</td></tr> <tr><td>13</td><td>12288</td></tr> </tbody> </table>	x	f(x)	10	1536	11	3072	12	6144	13	12288	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td>16</td><td>98304</td></tr> <tr><td>17</td><td>196608</td></tr> <tr><td>18</td><td>393216</td></tr> <tr><td>19</td><td>786432</td></tr> </tbody> </table>	x	f(x)	16	98304	17	196608	18	393216	19	786432
x	f(x)																					
10	1536																					
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12	6144																					
13	12288																					
x	f(x)																					
16	98304																					
17	196608																					
18	393216																					
19	786432																					

Figure 17: Tabulating a geometric sequence

As Figure 17 shows, users can scroll the table to see readily that the 12th term is 6144 and the first term larger than six lakhs is $T_{19} = 7,86,432$.

Series are a little more problematic on calculators, as the formulas involved tend to be quite complex. An easier approach on *ClassWiz* is to use a special summation function (Σ) that is available on the keyboard for evaluating series. For practical purposes, however, a table is a useful tool for users to see a series as a sequence of partial sums, as illustrated in Figure 18 for the same sequence with $T_n = 3 \times 2^{n-1}$, $n = 1, 2, 3, \dots$. In the second screen, note that the series is defined as $g(x)$ using the Σ function available on *ClassWiz*.

$f(x) = 3 \times 2^{x-1}$	$g(x) = \sum_{x=1}^x (3 \times 2^{x-1})$	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> <th>g(x)</th> </tr> </thead> <tbody> <tr><td>1</td><td>3</td><td>3</td></tr> <tr><td>2</td><td>6</td><td>9</td></tr> <tr><td>3</td><td>12</td><td>21</td></tr> <tr><td>4</td><td>24</td><td>45</td></tr> </tbody> </table>	x	f(x)	g(x)	1	3	3	2	6	9	3	12	21	4	24	45
x	f(x)	g(x)															
1	3	3															
2	6	9															
3	12	21															
4	24	45															

Figure 18: Tabulating a sequence and its associated series

As for the case of sequences, successive terms of the series can be read directly from the table as values of the function g . In the third screen shown in Figure 18, note that $g(4) = 3 + 6 + 12 + 24 = 45$, the fourth term of the series.

While algebraic formulas are helpful for resolving many problems, the calculator tables allow students to see several terms of the sequence or series at the same time and hence to potentially see them in better context.

Limits and convergence

The idea of a limit is often regarded as an important one in early studies in school calculus (although it is worth noting that neither Newton nor Leibniz used it explicitly in developing calculus). A key idea is that of getting 'closer and closer' to a value, without ever actually reaching it. On a calculator with table capabilities, successive values can sometimes be represented in a table, to help users see the limiting processes involved.

Consider for example the fundamental trigonometric limit, $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. To study the behaviour of this limit with a table, a suitable function first needs to be defined, after *ClassWiz* has been set to radian measure as shown in Figure 19:

$f(x) = \frac{\sin(x)}{x}$	Table Range Start :-0.1 End :0.1 Step :0.01	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td>9</td><td>-0.02 0.9999</td></tr> <tr><td>10</td><td>-0.01 0.9999</td></tr> <tr><td>11</td><td>0 ERROR</td></tr> <tr><td>12</td><td>0.01 0.9999</td></tr> </tbody> </table>	x	f(x)	9	-0.02 0.9999	10	-0.01 0.9999	11	0 ERROR	12	0.01 0.9999
x	f(x)											
9	-0.02 0.9999											
10	-0.01 0.9999											
11	0 ERROR											
12	0.01 0.9999											

Figure 19: Using a table to explore a trigonometric limit

A table of values generated on a small interval close to $x=0$ gives some insight into the limiting process. The calculator recognises that the expression is undefined at $x=0$, but the tabulated values make it clear that the values seem to get closer to the limit of 1 as the x -values approach zero from either side. Users scrolling the table of values can see for themselves the 'closer and closer' idea very well, as shown in Figure 20.

Table Range Start: -0.01 End : 0.01 Step : 0.001	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>10 -1 × 10⁻³</td> <td>0.9999</td> </tr> <tr> <td>11 0</td> <td>ERROR</td> </tr> <tr> <td>12 1 × 10⁻³</td> <td>0.9999</td> </tr> <tr> <td>13 2 × 10⁻³</td> <td>0.9999</td> </tr> </tbody> </table>	x	f(x)	10 -1 × 10 ⁻³	0.9999	11 0	ERROR	12 1 × 10 ⁻³	0.9999	13 2 × 10 ⁻³	0.9999
x	f(x)										
10 -1 × 10 ⁻³	0.9999										
11 0	ERROR										
12 1 × 10 ⁻³	0.9999										
13 2 × 10 ⁻³	0.9999										

Figure 20: Examining values ‘closer and closer’ to 0

Users can readily define smaller and smaller intervals to do so, as shown in Figure 21. (Notice that columns are constrained in size, so that only the highlighted value is shown to full accuracy, so scrolling is necessary.)

Table Range Start: -1 × 10 ⁻³ End : 1 × 10 ⁻³ Step : 0.0001	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>9 -2 × 10⁻⁴</td> <td>0.9999</td> </tr> <tr> <td>10 -1 × 10⁻⁴</td> <td>0.9999</td> </tr> <tr> <td>11 0</td> <td>ERROR</td> </tr> <tr> <td>12 1 × 10⁻⁴</td> <td>0.9999</td> </tr> </tbody> </table>	x	f(x)	9 -2 × 10 ⁻⁴	0.9999	10 -1 × 10 ⁻⁴	0.9999	11 0	ERROR	12 1 × 10 ⁻⁴	0.9999	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>10 -1 × 10⁻⁴</td> <td>0.9999</td> </tr> <tr> <td>11 0</td> <td>ERROR</td> </tr> <tr> <td>12 1 × 10⁻⁴</td> <td>0.9999</td> </tr> <tr> <td>13 2 × 10⁻⁴</td> <td>0.9999</td> </tr> </tbody> </table>	x	f(x)	10 -1 × 10 ⁻⁴	0.9999	11 0	ERROR	12 1 × 10 ⁻⁴	0.9999	13 2 × 10 ⁻⁴	0.9999
x	f(x)																					
9 -2 × 10 ⁻⁴	0.9999																					
10 -1 × 10 ⁻⁴	0.9999																					
11 0	ERROR																					
12 1 × 10 ⁻⁴	0.9999																					
x	f(x)																					
10 -1 × 10 ⁻⁴	0.9999																					
11 0	ERROR																					
12 1 × 10 ⁻⁴	0.9999																					
13 2 × 10 ⁻⁴	0.9999																					

Figure 21: Further refinement to get ‘closer and closer’ to a limit

Together, these capabilities allow users to appreciate that the important result that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ at least seems to be reasonable.

A similar process can be used to explore limits to infinity, although of course the (finite) calculator can only approximate the concept of (unbounded) infinity. To illustrate this, consider the infinite limit, $\lim_{n \rightarrow \infty} \frac{(5n + 7)}{2n - 11}$. To represent the idea of a number increasing without bound, a calculator like *ClassWiz* is restricted to choosing larger numbers and inferring what will happen as they increase. While imperfect, this process still offers insights. In this case, the function needs to be defined using x instead of n , which is unlikely to be problematic for students. In the example shown in Figure 22, the range is from five to six lakhs – a very long way short of infinity, but sufficient to appreciate the limiting processes involved:

$f(x) = \frac{5x+7}{2x-11}$	Table Range Start: 500000 End : 600000 Step : 10000	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>8 570000</td> <td>2.5</td> </tr> <tr> <td>9 580000</td> <td>2.5</td> </tr> <tr> <td>10 590000</td> <td>2.5</td> </tr> <tr> <td>11 600000</td> <td>2.5002875</td> </tr> </tbody> </table>	x	f(x)	8 570000	2.5	9 580000	2.5	10 590000	2.5	11 600000	2.5002875
x	f(x)											
8 570000	2.5											
9 580000	2.5											
10 590000	2.5											
11 600000	2.5002875											

Figure 22: Using a table to explore an infinite limit

Scrolling the table shows that the limit seems to be approaching 2.5 as the values of the independent variable increase. (Notice that the value is actually shown as 2.5, unless it is highlighted to show more precision.) Of course, much larger values can be used – with due care that the interval does not result in a table with too many entries. With such a capability, users can appreciate the reasonableness of the result that $\lim_{n \rightarrow \infty} \frac{(5n + 7)}{2n - 11} = \frac{5}{2}$.

A celebrated infinite limit is that used to define the exponential constant, $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$. This can be studied on the calculator in a similar way to other infinite limits, as shown in Figure 23, using a range for the variable of five to six lakhs, as previously.

$f(x) = \left(1 + \frac{1}{x}\right)^x$	Table Range Start: 500000 End : 600000 Step : 10000	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>8 570000</td> <td>2.7182</td> </tr> <tr> <td>9 580000</td> <td>2.7182</td> </tr> <tr> <td>10 590000</td> <td>2.7182</td> </tr> <tr> <td>11 600000</td> <td>2.718279552</td> </tr> </tbody> </table>	x	f(x)	8 570000	2.7182	9 580000	2.7182	10 590000	2.7182	11 600000	2.718279552
x	f(x)											
8 570000	2.7182											
9 580000	2.7182											
10 590000	2.7182											
11 600000	2.718279552											

Figure 23: Exploring the exponential function

Although the result shown to four decimal places is close to the known result of e , notice that tabulated values are truncated rather than rounded, so only the first three decimal places are correct. This limit converges very slowly, so this is not a practical means of approximating e via a limit. A better means of doing so involves the use of the infinite series expansion for e :

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, -\infty < x < \infty$$

The tables in Figure 24 show how this series can be used with $x = 1$ to explore the limit much more efficiently. The second screen shows the first four terms of the series, while the third screen shows that the first nine places of decimals for e are obtained with remarkably few (twelve) terms of the series:

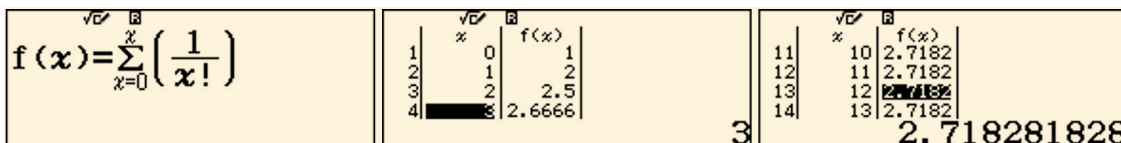


Figure 24: Exploring an infinite series for the exponential function

In these kinds of ways, tables on calculators can be used to provide experiences of limiting processes and of convergence, without the user being required to undertake excessive by-hand computation.

Calculus

Like other advanced scientific calculators, *ClassWiz* provides both a numerical derivative and a definite integral command, both illustrated in Figure 25. (In each case, the first line shows the command entered, while the second line shows the result generated by the command.)

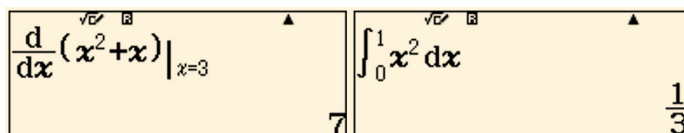


Figure 25: Calculus commands on *ClassWiz*

Once again, the availability of tables on a calculator allows users to understand important features of the mathematical concepts involved. The numerical derivative of a function $f(x)$ at a point $x = a$ can be understood through a limiting process via the First Principles definition, as suggested in the previous section:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

However, a powerful use of the table facility is to evaluate a succession of derivatives at a point and to see the emerging patterns involved. To illustrate with an unsophisticated example, often the first one studied by calculus students, consider the calculator screenshots shown in Figure 26. The first screen displays the expression for the derivative of the x^2 function at a given point (named as $f(x)$). The second screen displays the numerical values of this function.

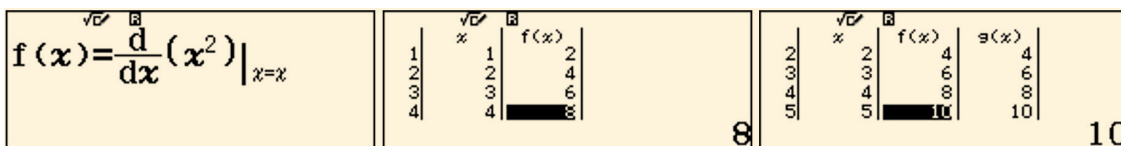


Figure 26: Using tables to represent a derivative function

Students studying the calculus, should have no difficulty in seeing the extraordinary pattern in the numerical derivatives that are shown in the table, with each value twice the value of the independent variable. Scrolling the table will allow them to predict the next values and affirm their understanding of the pattern, as suggested by the third screen above. Students are likely to find this insightful in making the conceptual leap from the idea of a derivative at a point to the more powerful idea of a derivative *function*, in this case $g(x) = 2x$, shown in the third screen in Figure 26.

Tables can be used in a comparable way, albeit limited, to see the general situation associated with integration, rather than the specific situation of an individual definite integral. This is illustrated in Figure 27, again in the case of an unsophisticated example, nonetheless of help to novice students.

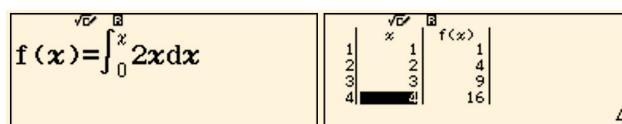


Figure 27: Exploring patterns in integration

In this case, as in the previous one, the pattern of results seems to be very clear, and likely to be recognised by novice students, helping them to get some insight into the general result $\int 2x dx = x^2$, a powerful instance of the relationship between differentiation and integration, ultimately reflecting the Fundamental Theorem of Calculus.

These results, of course, need further experiences for sound learning, and it is not suggested that a calculator like *ClassWiz* by itself is sufficient. However, it is suggested that the calculator – through its table facilities – offers opportunities for learning that are easily overlooked, especially if the calculator is regarded as merely a computational device to get numerical answers.

Conclusion

An advanced scientific calculator like *ClassWiz* is better understood as a device to provide students with experience than as a device to provide students with answers. The availability of tables on modern calculators opens up many opportunities for doing this, well reflected in the proposed model and showing how representation, exploration and affirmation are at least as important for learning as is computation. This paper has shown several examples of these relevant to typical secondary school curricula. Tables can be used to represent concepts such as functions, identities, equations, limits, convergence and derivative functions. They allow students to undertake personal explorations related to these concepts, to understand better the mathematics involved. Well used, the table features on calculators allow students to make and test their own conjectures, with opportunities to affirm these or to contradict them, supporting their learning in either case. The examples offered in the paper are illustrative, not exhaustive: students and teachers can find many more examples for themselves using advanced scientific calculators with a capability to generate and productively use tables of values.

References

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