MEASURING EARTH'S SIZE

AMOL ANANDRAO KATE

When was the first accurate measurement of the earth's size made? Who were the first people to make this measurement? How did they make it?

It is widely believed that the earliest efforts to measure the size of the earth coincided with the first voyages on the high seas. However, we now know that the first near-accurate measurement of the size of the earth was made much earlier (and more than 2000 years ago) by a Greek called Eratosthenes. Eratosthenes accomplished this feat at a time when Europe had not yet discovered continents like America, Australia, and Antarctica, or details of the Pacific, Atlantic, or Indian Oceans (see **Fig. 1**).

Who was Eratosthenes?

Eratosthenes was a scholar and philosopher, who was born in 276 BC in a town named Cyrene that was then part of the Greek Empire. This place is today known as Shahat, and is part of Libya. After an early education in his home town, Eratosthenes travelled to Athens to study philosophy and poetry. At the age of thirty, Ptolemy III, the ruler of Alexandria, in Egypt, offered Eratosthenes the position of a librarian in The Great Library of Alexandria, one of the largest and most important libraries in the ancient world. Eratosthenes accepted this offer and moved to Alexandria, where he lived till his death in 194 BC. During this period, he rose to the position of Chief Librarian, expanded the library's collection of books, and tutored Ptolemy's children.

Eratosthenes was a polymath. Not only did he make many important contributions to mathematics and science, he was well versed in subjects like geography, literature, poetry, and history. Much of his work has been lost to the ages, but we know of some of his contributions through the writings of other philosophers

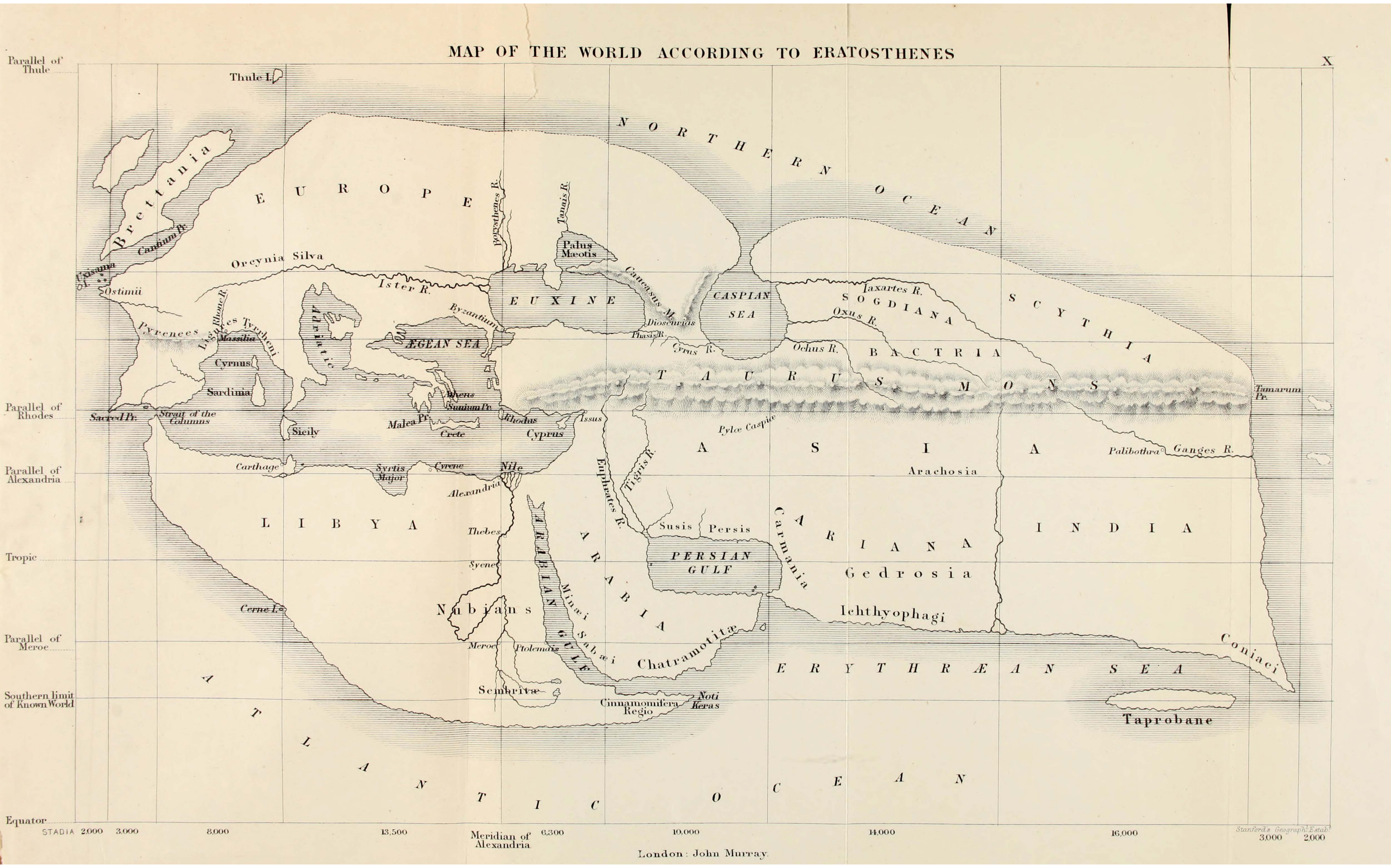


Fig. 1. **A 19th century reconstruction of Eratosthenes' map of the known world.**

Credits: Bunbury, E.H. (1811-1895), A History of Ancient Geography among the Greeks and Romans from the Earliest Ages till the Fall of the Roman Empire, page 667. London: John Murray, 1883. Uploaded by Sette-quattro on Wikimedia Commons. URL: https://commons.wikimedia.org/wiki/File:Mappa_di_Eratostene.jpg. License: CC-BY.

like Cleomedes, Pappo, and Strabo. In fact, it is Cleomedes who shares a simplified version of the method used by Eratosthenes to measure the circumference of the earth.

How did Eratosthenes measure the size of the earth?

By this time, it was widely believed that the earth was shaped like a sphere. Many people had tried to measure its circumference, but had failed. Eratosthenes used his knowledge of geometry to arrive at this measurement. He knew that objects in a town called Syene, which was to the south of Alexandria, seemed to cast no shadow at noon of a summer solstice. One could see the sun's rays shining straight down a well — its reflection fell only on the water at the bottom, and did not extend to the sides. This was because the sun was directly above (at a 90° angle) Syene at this time. Syene, now known as Aswan, is located very near the Tropic of Cancer, where the sun is directly overhead at noon around 21st June, the summer solstice of the Northern Hemisphere.

In contrast, Eratosthenes had noticed that even when the noon sun of the summer solstice in Alexandria looked as if it were directly overhead, objects around him, including a pole that he had erected, cast longer shadows than the ones he had seen in Syene (see **Fig. 2**). This difference in the length of shadows meant that the rays of the sun fell at an angle at Alexandria. He surmised that he could measure the size of the earth if he knew the distance between Alexandria and Syene, and could measure the angle by which the rays of the sun at Alexandria deviated from those at Syene. How? In the same way that we can determine the perimeter of a circle if we know the angle subtended by an arc of the circle, and the length of the arc for that angle.

Eratosthenes used a stick to measure the angle by which the rays of the sun deviated from the normal at

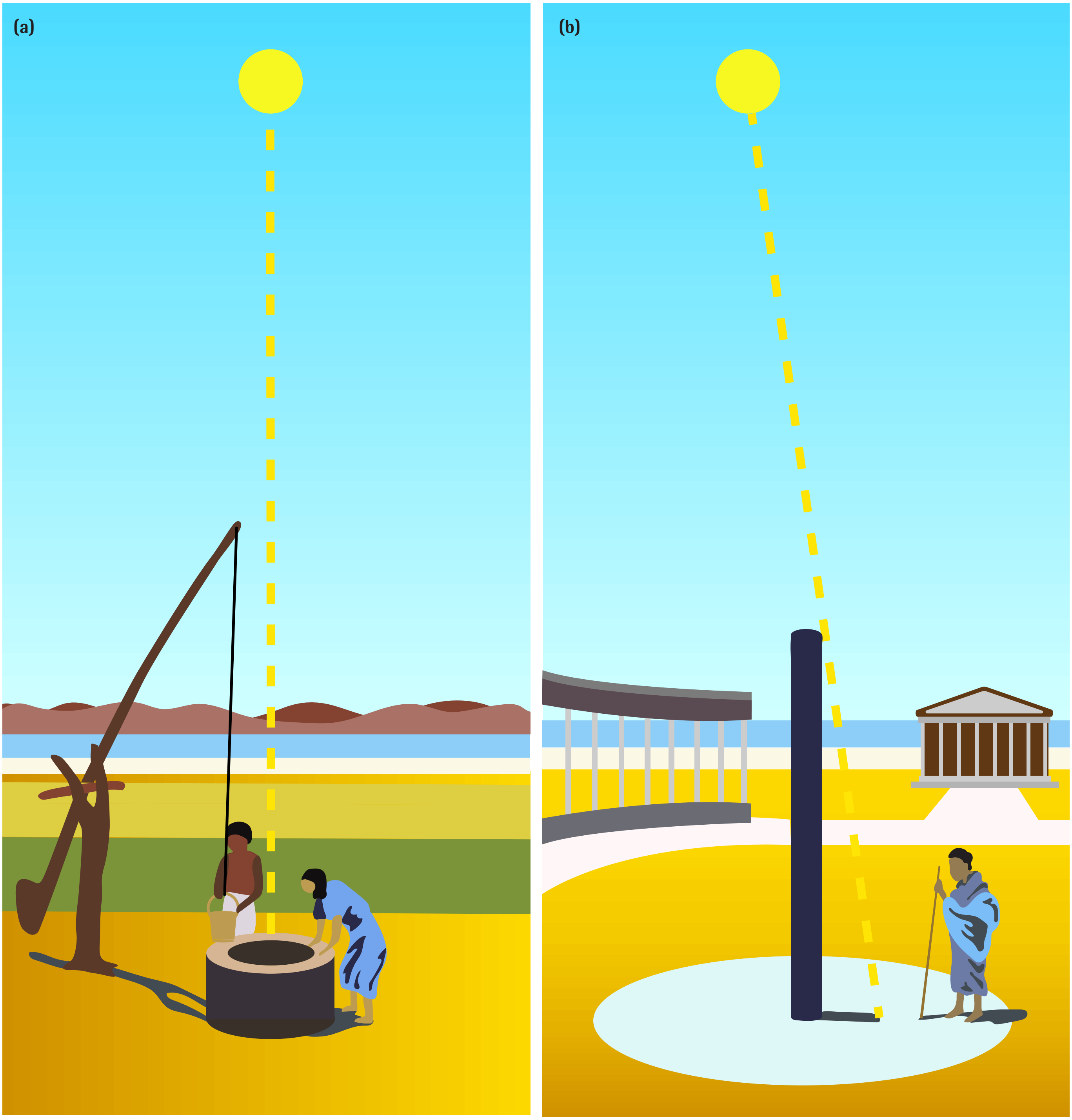


Fig. 2. Eratosthenes knew that the angle of the shadow under the noon sun on the day of the summer solstice in the Northern Hemisphere was different in (a) Syene and (b) Alexandria.

Adapted from http://karidiscience.blogspot.com/2013/03/blog-post.html. Credits: Agios Nikolaos on 'Karydis Manos'. License: CC-BY-NC.

Alexandria, and found it to be 7.2 degrees. Eratosthenes hired someone to measure the distance between Syene and Alexandria, which he learnt was about 5000 stadia (a Greek measure of distance that equalled about 157 m in Egypt). This distance was the arc of the earth's circle that was equivalent to an angle of 7.2 degrees (see **Fig. 3**). By comparing the length of the arc for this angle with that of the full circle, Eratosthenes calculated the circumference of the earth to be about 250,000 stadia (see **Box 1**). In other words,

The circumference of earth/ Distance between Alexandria and Syene = 360°/ angle subtended by the arc made by the two cities at the centre of the earth.

The distance between Alexandria and Syene = 5000 stadia.

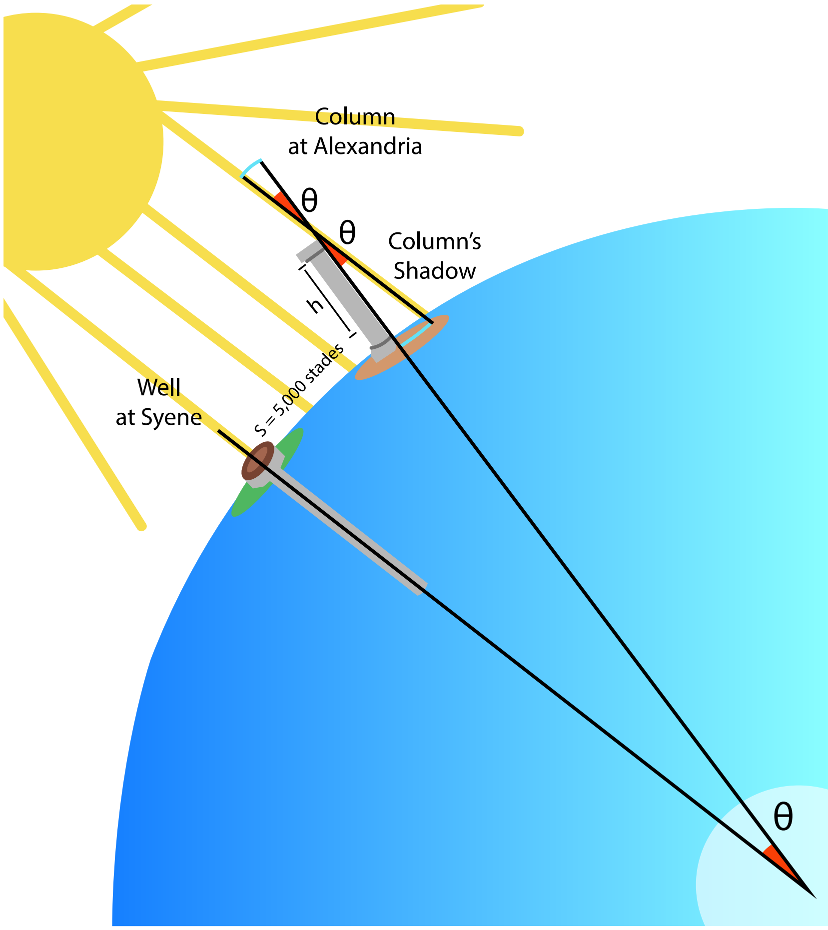


Fig. 3. Measuring the angle by which the rays of the sun at Alexandria deviated from those at Syene at noon on summer solstice. Eratosthenes made one assumption — because the sun was so far away, the rays of the sun would fall parallelly on earth. He measured the angle at which the rays of the sun fell upon a stick in Alexandria to be 7.2 degrees. Based on his knowledge of geometry, he knew that this was an alternate (interior) angle to the angle that the distance between Alexandria to Syene subtended at the centre of the earth. Thus, both angles would be equal.

Adapted from https://www.flickr.com/photos/nasablueshift/9411406224. Credits: NOAA Ocean Service Education. License: CC-BY.

Box 1. Try Eratosthenes' method for yourself

Isn't it remarkable that Eratosthenes was able to arrive at such an accurate measurement with just a stick and some elementary geometry? If you and your students would like to try his method out for yourselves, check out our **Activity Sheet** titled: **How do we measure the size of the earth?**

The angle subtending the distance between the two cities (measured by him) = 7.2°.

Therefore, the circumference of earth/5000 stadia = 360°/7.2°.

This meant that the circumference of earth = 50 × 5000 = 250,000 stadia.

The metric equivalence of Eratosthenes' measurement of the earth's circumference would be 250,000 stadia × 157 m = 39,250,000 m or 39,250 km. This is very close to the value we have today (40,075 km)!

Key takeaways

- The first near-accurate measurement of the size of the earth was made more than 2000 years ago by a Greek called Eratosthenes.
- While much of Eratosthenes' work is lost to the ages, we know of the method he used to arrive at this measurement from the writings of the philosopher Cleomedes.
- Eratosthenes was able to arrive at an accurate measurement with just a stick and some elementary geometry. Any of us can replicate this method to measure the earth's size for ourselves.

Notes:

1. Eratosthenes' method was described by Carl Sagan in this video: https://www.youtube.com/watch?v=G8cbIWMv0rI.
2. Source of the image used in the background of the article title: 'Eratosthenes Teaching in Alexandria' by Bernardo Strozzi. Credits: mark6mauno, Flickr. URL: https://www.flickr.com/photos/mark6mauno/10832052985. License: CC-BY.

Amol Anandrao Kate works with the Azim Premji Foundation as a teacher educator and science resource person at Sirohi, Rajasthan. He can be contacted at: amol.kate@azimpremjifoundation.org.

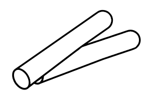
ACTIVITY SHEET : HOW DO WE MEASURE THE SIZE OF THE EARTH?

You can use this simple activity to measure the circumference of earth for yourself. To do this, you will need to know the shortest distance of your city/town/village from the equator. Your task will become easier on the two days of each year when the sun is directly overhead at high-noon at the equator — 20th March and 21st September. Since this method involves measuring angles and comparing numbers, you will need the math you learned in middle school.

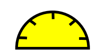
You will also need:

A metre scale, or a straight stick that is a metre in length.

A flat surface on which the meter stick can be placed at an angle of 90°. Make sure that this surface receives sunlight.

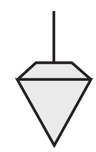


A piece of chalk with which to mark the shadow of the stick.

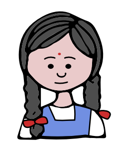


A protractor

A long thread



A plumb line



A companion to measure the length of the shadow

What to take care of:

- Pick a day when the sky is clear. If it is cloudy, you may not see a shadow. If you are doing this activity in March, the sky is generally clear in most parts of India. If you are doing this activity in September, and the sky is cloudy, you can postpone it by a day or two.
- Use a calendar or almanac to find out in advance the time of the day when the sun will be directly overhead in your location (solar noon).
- Keep all the equipment ready 10 minutes prior to the actual activity.

What to do:

1. Set up the meter stick on a flat surface that receives direct sunlight. The stick should be vertical to the surface — you can check this with a plumb line.

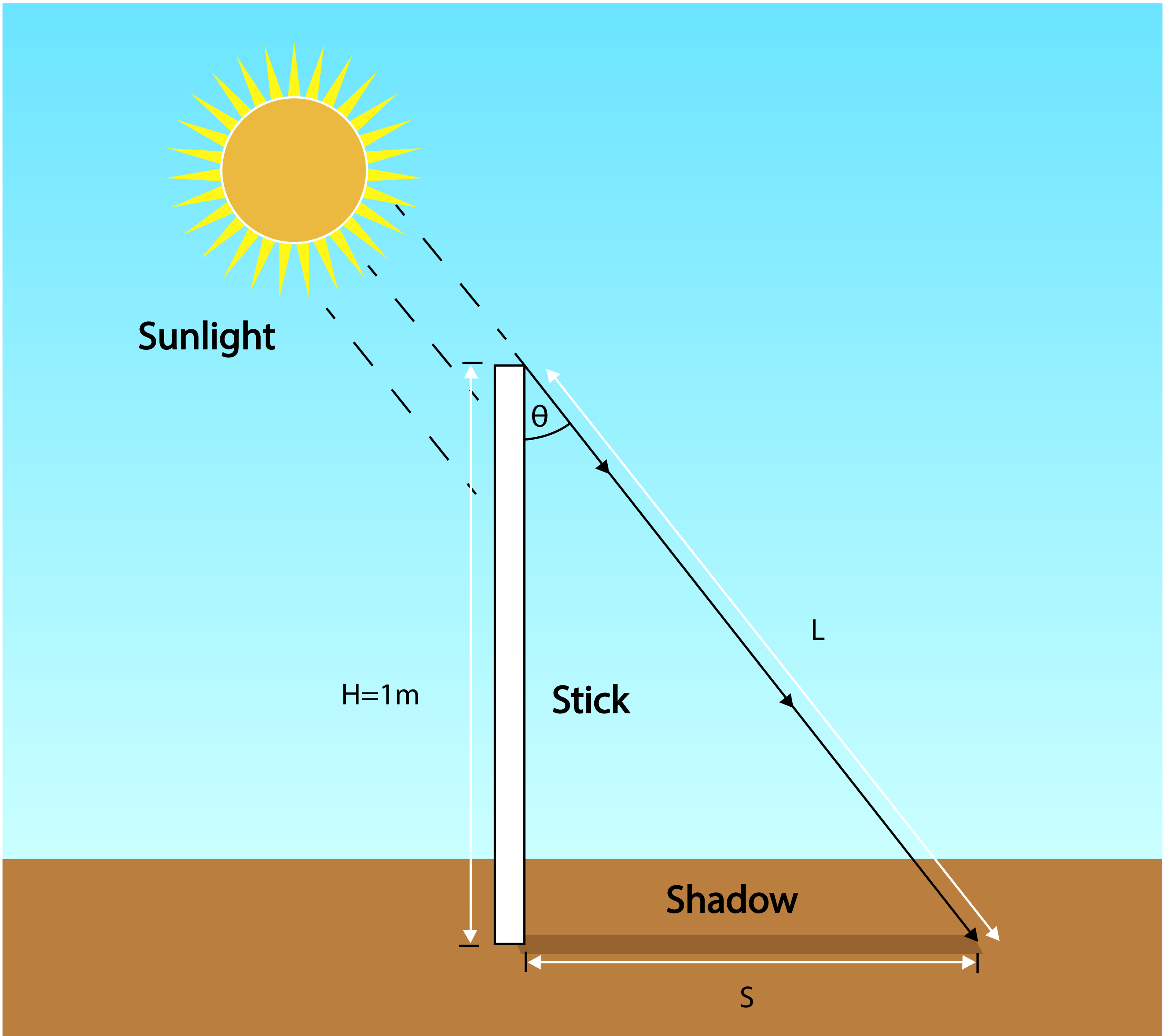
2. When the sun is directly overhead (highest in the sky), use the chalk piece to mark the length of the shadow of the stick that falls on the flat surface. In India, this shadow will be towards the north of the stick. (Why? Find out.)

3. To measure the angle at which sunlight falls on the stick, stretch a thread from the top of the stick to the tip of its shadow with your companion's help.

4. Also measure the angle between the thread and your stick with a protractor. Note it down.

5. You can use the distance of your location (town or city) from the equator to calculate the circumference of the earth using the following formula:

$$\frac{\text{360 degrees}}{\text{Angle of the shadow}} = \frac{\text{Circumference of the earth}}{\text{Distance of your location from the equator}}$$



Credits: Adapted from an image by Dr. Angelos Lazoudis, Ellinogermaniki Agogi, Research & Development Department, Pallini, Greece.
URL: https://eratosthenes.ea.gr/content/experiment.

Think about:

- If you were to use this method to measure the circumference of the earth on different days, would the value you get be different? Why? Can you design an experiment to test your hypothesis?
- If we wanted to use this method on some other day of the year, what changes do you think we'd need to make to it?

Contributed by:
Amol Anandrao Kate, who works with the Azim Premji Foundation as a teacher educator and science resource person at Sirohi, Rajasthan. He can be contacted at amol.kate@azimpremjifoundation.org.

Ask a Question

ACTIVITY SHEET: HOW DO WE MEASURE THE SIZE OF THE EARTH?

TEACHER'S GUIDE

On 20th March and 21st September, the sun is directly overhead at any point on the equator. All of India is North of the equator and the latitude varies from 804' North of the equator at the southernmost point on the mainland to 3706' North of the equator at the northernmost point. All locations in our country fall within this range. This means that when the sun is directly overhead at the equator on one of these two days, a location in India at the same longitude will see the sun at an angle to the South of that vertical. In other words, at 12 noon at any location in our country, the sun would be at the highest point in its trajectory of East to West, but at an angle to the South. If we know this angle, and the distance of that location to the equator, calculating the circumference of the earth is simple.

However, depending on where we are in India in terms of longitude, the sun may not be exactly at the highest point of the trajectory at 12 noon. Indian Standard Time (IST) is the time at 82.5 degrees East of Greenwich. The sun would be at its highest point in the trajectory at 12 noon IST on all points at this longitude. At locations to the East of this, the sun would be at the highest point in the trajectory earlier than 12 noon; and at locations west of this longitude, later than at 12 noon. An almanac can tell us at what time this will happen. Alternately, if one can identify one's location on a map and its longitude, one can calculate at what time the sun will be at the highest point in that location. How? Since each degree East will add 4 minutes and each degree West will reduce 4 minutes, multiply the difference in longitude (from 82.5 degrees) by 4. Add or subtract the product from 12 noon depending on whether one is East or West of the equator.

Contributed by:
Ramgopal (RamG) Vallath, who is a motivational speaker. He is also the bestselling author of the children's science fiction, 'Oops the Mighty Gurgle'. He delivers motivational talks and science workshops in schools. He can be contacted at ramg@azimpremjifoundation.org.

Azim Premji University