Designing a Good Maths Worksheet

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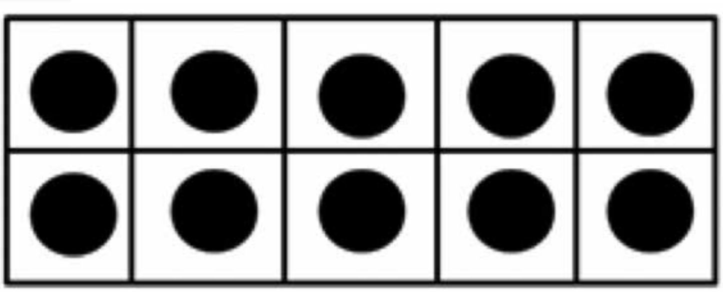
In my experience as a maths teacher, I have found that students are sharply divided into two mutually exclusive sets – those who love maths and those who do not. There are no neutrals here and the reception accorded to a maths worksheet is similarly highly polarised. There are those who seize it, relish every challenge it provides them and submit it with happy anticipation of more and then there are those who gloomily sit over it, delaying putting pencil to paper and submit it with dread, knowing that every mistake will be marked and made much of with the teacher's pen and returned without the student even beginning to understand what he/she was expected to do.

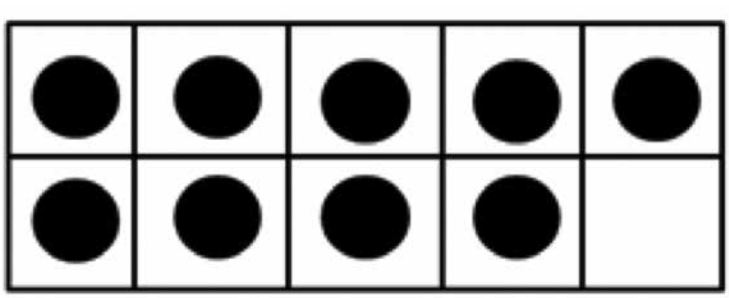
How can we move the student from the second category to the first? Secondly, how can we change the treatment given by the teacher to a student's responses to a worksheet? Can a worksheet be an evangelical tool which changes a student's attitude to maths? In its present avatar, a worksheet is most often seen as providing much-needed drill and practice of a particular concept. While I do not contest the need for this, I would like to suggest some ways in which a worksheet becomes an invitation for a student to enjoy doing maths. I use as my example a worksheet on addition and subtraction. The ideas may be extended to multiplication and division and in fact, some of the problems that are used set the foundation for the same.

A very useful foundational step for applying the algorithm for addition is understanding the decomposition of 10 and where else to find a helping hand than at the end of your own arms! From our fingers to tens frame makes for an organic progression and tens frames are easy to replicate in worksheets.

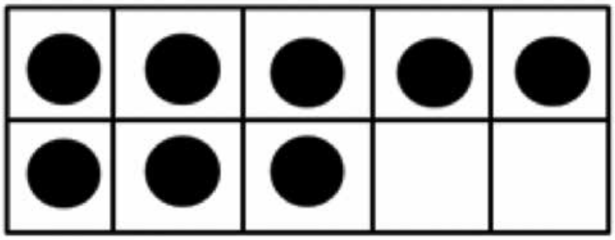


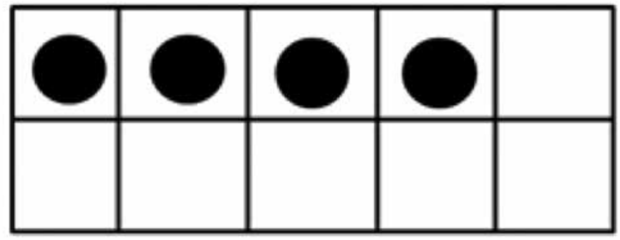
A tens frame is a simple 2 × 5 grid of dots
Removing any/some dots leads to powerful learning.
Here are some questions for the beginner's level:

For each of the following tens frames, write down the addition fact you see. Make a subtraction fact from it. The first one is done for you:



9 + 1 = 10 10 – 1 = 9 10 – 9 = 1





An important part of the symbolic representation of number statements such as

9 + 1 = 10; 10 – 1 = 9 and 10 – 9 = 1 is the conceptual understanding of the meaning of these statements. Hence, a follow-up question could be to write or draw a story using a particular number statement. A student who is able to show a set of 9 pencils and then add 1 pencil to it has a deep understanding of the numerals 9, 1, 10 and the '+' and '=' signs. The use of the tens frame also helps the students to visualise the different contexts in which addition is used:

- Increasing a number by another quantity (I had 9 pencils and I was given 1 more pencil)
- Finding the required amount to raise a given number to a higher number (I have 9 pencils, how many more should I add to get 10 pencils)
- Combining two groups (one pencil box had 9 pencils and the other had 1 pencil, how many pencils in both pencil boxes altogether).

Once the student has mastered addition facts for 10, pairs of tens frames may be used to add one-digit numbers. For example, finding 8 + 4. Most students tend to use counting on, but their practice of completing the ten will help them to break down the sum into 8 + 2 + 2 and this helps them to arrive at the total of 12 much faster. This may be encouraged by giving multi-step problems, such as

7 + 5 = 7 + ___ + __ = 10 + ___= ___

The practice of this sort of problem may be done using the tens frame and then abstracting the students' understanding with a worksheet using hops on a simple number strip (Figure 1).

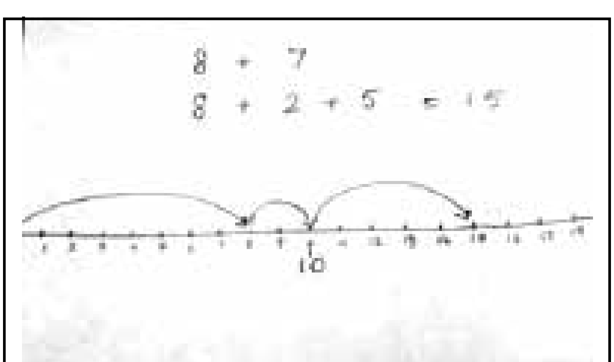


Figure 1. Source P Shirali; Addition Pullout: At Right Angles, July 2013

Here, the worksheet plays the important role of consolidating students' understanding and building the foundation to help them understand two-digit addition problems later. Of course, before launching into the algorithm, the student should have practice in adding multiples of ten and in using bundles and sticks to visualise the 'carrying over' process (Figure 2).

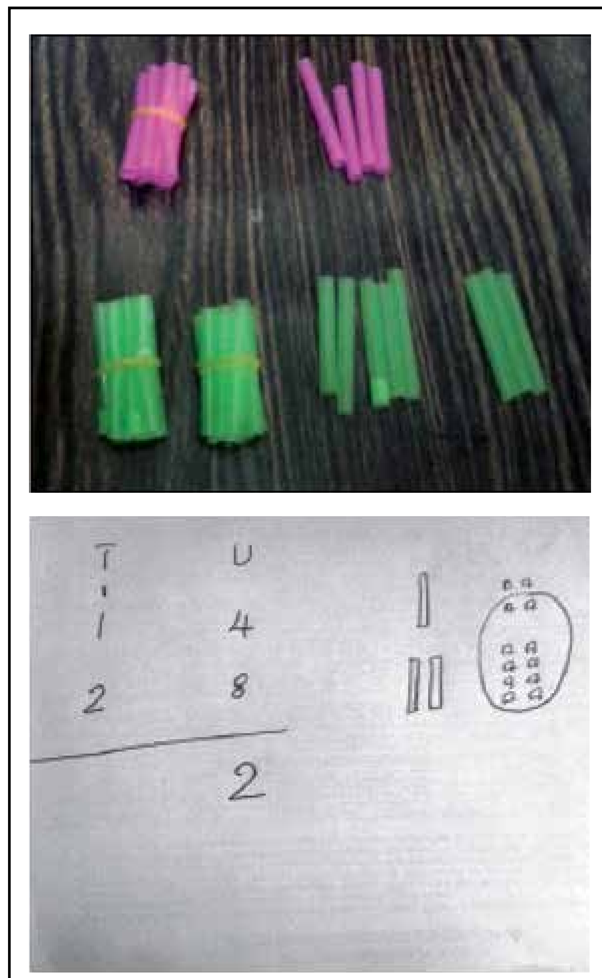


Figure 2. Source P Shirali; Addition Pullout: At Right Angles, July 2013

Notice the documentation of the activity. This is very important – most worksheets have multiple problems on column addition but bridging the gap between activities and algorithms is often neglected and worksheets are a useful tool to do this. Taking a cue from Figure 2, the teacher can design multiple questions using a three-column format.

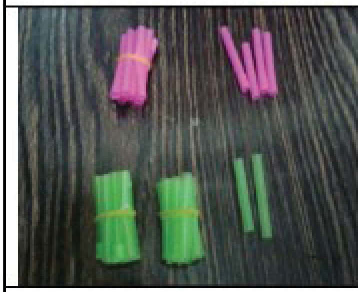
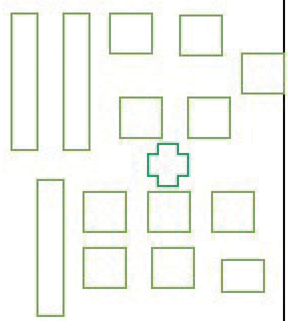
<table>
<tr><th>Column 1</th><th>Column 2</th><th>Column 3</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td><table><tr><th>TENS</th><th>ONES</th></tr><tr><td>3</td><td>5</td></tr><tr><td>4</td><td>7</td></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td colspan="3">Fill in the three columns when the sum is 87</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Table 1. Three-column format to design questions

Questions can be open-ended if only the sum is given (as in row 4), students can also design their own problems once they have gained confidence.

This may be extended to multi-digit addition without the use of manipulatives when the student has understood the meaning and value of the number carried over.

A similar approach to subtraction will help the learner not only understand the meaning and process of subtraction but will also prepare him or her to understand clearly when to apply which operation. The contexts in which subtraction are used are:

- Subtraction as a take-away (removing from a pile)
- Subtraction as a comparison (how many more/ how much taller)
- Subtraction as the inverse of addition (how many to be added)

Worksheets on subtraction should make use of the *ganitmala* with students observing all three contexts by moving backwards and forward and by finding the number of beads between two given beads. Again, this may be reinforced with worksheet problems using hops on the number strip.

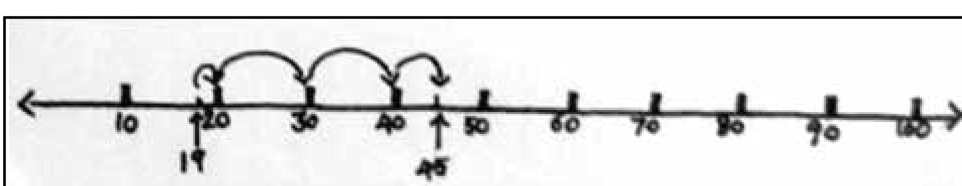


Figure 3. Source P Shirali; Subtraction Pullout: At Right Angles, November 2013

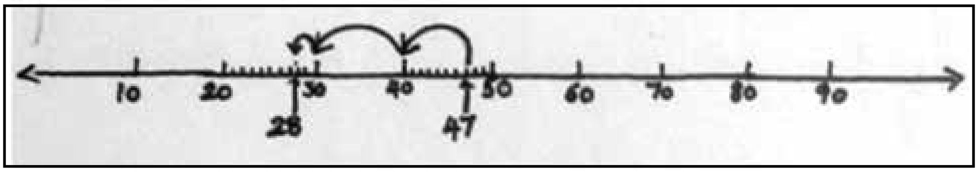


Figure 4. Source P Shirali; Subtraction Pullout: At Right Angles, November 2013

Here, the hops are forward, the corresponding question in the worksheet could be based on the third context and may be phrased as: How many to be added to 19 to get 45? Scaffolding may be provided by initially marking the points 19 and 45 on a graduated number strip and expecting the student to draw only the hops. Later problems may require students to mark the two given points and finally, even graduate the number strip before solving the problem.

Backward hops may be used to frame questions in the second context: By how much is 28 less than 47?

By the end of the worksheet, the student should be able to grasp that subtraction is the difference between two numbers and draw out or write short word problems which include the first context of subtraction i.e., subtraction as a take-away (removing from a pile).

Worksheets usually point towards accurate answers, but this means that a student is either right or wrong. Adding questions which estimate the answer helps students recognise that the answer they have got is wrong and attempt to correct themselves. For example, in the above example, a preliminary question on 'the difference between 28 and 47 is between ___ and ___ may be easily estimated by the student by actually seeing that 47 -28 is between 48–28 and 38–28. Such questions also provide invaluable insight to the teacher regarding the student's conceptual understanding and thus, help in formative assessment and remedial teaching.

By class IV, when students do addition and subtraction of four-digit numbers, it is important for them to practise again on the open number line by moving to the nearest ten, the nearest hundred and the nearest thousand. This helps them negotiate difficult problems, such as 2415 -1099 and in fact, develop their own shortcuts to finding the difference. A worksheet should have the space for the student to describe how they solved the problem; they may be given the opportunity to draw or state in point form how they got the solution. This helps students to not feel constrained to follow a standard algorithm but exercise their own conceptual understanding to arrive at the solution. Further, worksheets which relate the algorithm for column addition to the hops on the number line help to move them to competence in this skill.

When students reach the stage of requiring drill and practice once they have demonstrated conceptual understanding, a problem like this has interesting spin-offs.

Wahida made 104 *laddus* for Eid to distribute to her eight neighbours.

She laid out 8 trays and put 2 *laddus* in each tray. How many laddus did she have left?

She repeated this till all the *laddus* were over. Show how many *laddus* were left with her at each stage.

How many *laddus* did each neighbour get?

Can you suggest a quicker way for Wahida to distribute the *laddus*?

Clearly, this repeated subtraction is paving the way for division. Notice that such a problem can be set even if the student has only learnt addition or subtraction. By asking the student for a quicker way, the problem gives the child scope to appreciate the long division algorithm or at least the thinking behind it, when they do learn it in the later years. Careful facilitation of the discussion of the answers provided by different children can help the teacher arrive at 10 added 8 times (or 8 × 10 if they have done multiplication) = 80 and 108-80 = 24 which gives each neighbour three more *laddus*, bringing the total share to 10 + 3 laddus. This is exactly the reasoning followed by the long division algorithm. Such an introduction helps students to circumvent problems such as the zero in the middle of the dividend (which can lead them to say that 404 divided by 4 is 11 if they blindly follow the algorithm without understanding). Again, here questions on estimation will also help avoid such errors.

It also makes sense of the fact that division is the only arithmetic operation for which the algorithm begins with the highest place value (i.e., from the left).

Designing a good worksheet is, therefore, not just a matter of providing students with enough material to keep them occupied 'productively'. In summary:

- It starts with understanding what we mean by productive, in other words, the learning outcomes of the unit for which the worksheet is being designed have to be very clear to us.
- Figures and images are powerful tools in a worksheet and can be used to stimulate reasoning, dialogue and problem posing.
- Worksheets can provide a powerful bridge between the concrete and the abstract and following an activity, worksheets should provide practice of the abstracted version of the same activity.
- A worksheet should provide the opportunity to students to develop and hone their process skills of visualisation, representation, communication and estimation.
- Scaffolding questions which allow students to realise if their answer is wrong and attempt to backtrack and self-correct, provide powerful opportunities for self-assessment.
- Drill and practice of algorithms for arithmetic operations should be administered only when the teacher is certain that the student has a strong conceptual understanding of what has been taught.

How does the teacher harness the opportunities provided by a worksheet? It is important that the insights which can emerge from cleverly-designed questions are addressed in direct or group discussion so that the student understands his or her mistakes, if any. Making a portfolio of solved worksheets helps the student reflect on his or her progress and enables them to become aware of the careless mistakes or repeated errors which they tend to make Above all, the teacher should give the student ownership of the worksheet, this is the student's work and he or she should be given the opportunity and the insights to be proud of it.

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