1 A Culture of Enjoying Mathematics

Shashidhar Jagadeeshan

Introduction

It appears that whether we like it or not, Mathematics pervades all aspects of our lives. Whether you are a farmer or a techie, a comfortable relationship with Mathematics, and competency at the level at which one uses it, is a requisite in an equitable society. Some will argue that even if the content of Mathematics learnt at school is forgotten, students will retain the ability to think clearly and logically (an essential life skill) because of their exposure to mathematical reasoning. The tacit assumption here is that learning Mathematics will not only help us in our daily lives but will also enhance the quality of our life. How ironic that for a vast majority their experience with Mathematics is so contrary to this assumption. Enough has been written bemoaning the state of Mathematics education the world over, and the term 'Mathphobia' has become part of common parlance. A major reason for school dropout is the inability to cope with Mathematics; it seems to be a universal phenomenon that many students fear and dread Mathematics. Sadly, this feeling often persists into adulthood.

There have been many attempts to reform Mathematics education, and huge sums of money have been dedicated to this cause. Unfortunately, the motives for reform are suspect and, in my opinion, this is part of the problem. Advanced nations want to improve their citizens' Mathematics competency out of a fear that citizens of rival nations are outperforming them. Emerging nations want to improve their Matheducation so that they can create a 'knowledge society'. Humans empowered with knowledge are seen as a great asset in the market place. Reforms based on these motivations do not seem to have made much impact in the long run on Mathematics education (although there was a brief 'golden age of basic science' in the US thanks to the Sputnik fear).

If we are to make any headway in addressing both problems, that of poor mathematical competency and that of Mathphobia, we need to explore several questions first. What is the nature of Mathematics and how do our particular biases impact curriculum design? What is the relationship that students and teachers share with Mathematics? What are the myths or beliefs that studentst and teachers have about Mathematics? And perhaps most importantly, what are the factors that motivate humans to learn? In this article I hope to begin such an exploration by first describing the various ways in which Mathematics is viewed and experienced, and how these views might affect curriculum if applied in isolation. I then go on to look at curriculum design and pedagogy and see if we can truly create a culture of enjoying Mathematics not just for a elite few but for all.

The Blind Men and Mathematics

We are all familiar with the famous Jataka tale about the blind men and the elephant. Each one makes tactile contact with a different part of the elephant, and comes up with descriptions ranging from a wall to a rope! Mathematics too suffers from partial perceptions. Perhaps the mystery, depth and richness of Mathematics is revealed in the fact that it can be seen in so many different ways. Let us look at some of these perceptions and how they impact curriculum design and pedagogy.

Mathematics as accountancy: For a large majority of people, Mathematics is synonymous with accountancy. Perhaps it is not unreasonable to say that the bulk of humanity uses Mathematics to compare prices, make sure they are not being cheated of the correct change, perhaps calculate interests, discounts and rebates; some may even calculate areas and volumes. The more advanced may use it in book keeping. It is also true that many discoveries in arithmetic probably came from the need to keep records of land and accounts of trade. Examples that come to mind are preliminary trigonometry and mensuration, motivated by the need to calculate land holdings in the Nile Valley. Perhaps the motivation for the discovery of the Hindu-Arabic numeral system came more from the need to do large calculations in astronomy than from the need to do book keeping. But surely this discovery, considered one of the 'greatest intellectual feats of humans,' has had its major impact in the field of commerce. For most people arithmetic and number manipulation is synonymous with Mathematics.

If this is one's only experience with Mathematics, then one will design the curriculum and teach Mathematics as if it were a science of algorithms to do mechanical calculations and lose many students in this drudgery. Sofia Kovalevskaia expresses this wonderfully: "Many who have never had the occasion to discover more about Mathematics confuse it with arithmetic and consider it a dry and arid science. In reality, however, it is a science which demands the greatest imagination."

Mathematics as problem solving and mental gymnastics: One of the major features of Mathematics is problem solving, and many who discover the thrill of problem solving at a young age become professional mathematicians when they grow up. However, if this aspect of Mathematics is distorted, and seen in the wrong perspective, it becomes a source of fear and aversion toward Mathematics. Since talent in problem solving appears at a very young age, children are often classified as 'brilliant' or 'dull' based on this single ability. When an education system equates a child's self worth with their mathematical problem solving ability, it does great harm both to the ones who are adept at problem solving, and to those who are not. Those who find problem solving difficult, and who then go on to be labeled as 'stupid' (either by society or by themselves), develop a deep fear and aversion to all of Mathematics. This self image is often linked to their self esteem, leading to feelings of insecurity and shame. All of us have encountered perfect strangers who have a great need to confess how bad they are at Mathematics. On the other hand, those who are very adept at problem solving and Mathematics are automatically labeled 'intelligent', and run the risk of becoming one-dimensional human beings with poor social skills. I leave it as an easy exercise to name your favorite mathematician as an example to illustrate this point!

There is no doubt that a large part of mathematical theory is motivated by the desire to solve difficult problems. Fermat's Last Theorem is a famous example. However, not all mathematical problems are of the same order. Some problems are indeed very profound and like the tip of an iceberg, reveal deep aspects of Mathematics. Many problems (an endless plethora) are simply mental gymnastics often created by working backwards from solutions, requiring some inane trick to solve.

Those who find problem solving difficult, and who then go on to be labeled as 'stupid' (either by society or by themselves), develop a deep fear and aversion to all of Mathematics. This self image is often linked to their self esteem, leading to feelings of insecurity and shame.

These problems form the core of most of our competitive exams and are used as a sieve to weed out applicants. Any system that uses this gymnastic ability as a yardstick to decide how to distribute access to resources such as education and jobs will surely create a skewed society. The effects of these are already being seen today at our institutions of higher learning. Students who have been put through a grind of mindless problem solving are burnt out and have no motivation to learn anything new. Such students have a very narrow view of Mathematics and very few will choose Mathematics research and teaching as a career. I have heard senior professors and administrators bemoaning the fact that it is very hard to find competent people to teach Mathematics at many new prestigious institutions in India. Imagine the fate of the many thousands of students who have, after several years of preparation, failed to get access to so-called quality education. With damaged psyches and bruised self confidence, what kind of learning can take place? Further the erroneous identification our society has made between intelligence and mathematical ability has led to a dismal state of education for those interested in pursuing the humanities, because disproportionate funds are made available to science education. Many students with no real interest in the sciences and perhaps very gifted in other areas still pursue science.

Mathematics as the 'language of the universe' and as a useful tool in modern society: With Galileo, Mathematics has begun to be seen as the language of the

universe. Those who seek to unravel the mysteries of the universe see Mathematics as a sixth sense needed to comprehend the universe. We marvel with the physicist Eugene Wigner who gave a lecture in 1959 titled 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences.' Wigner ends his lecture by saying, "The miracle of the appropriateness of the language of Mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning." For many who pursue the more theoretical aspects of science, it is this 'miraculous' aspect that they most appreciate in Mathematics.

> *Mathematics with its brilliant ability to model phenomenon has had a far reaching impact in all aspects of our lives and on many fields of study, ranging from biology to economics.*

Mathematics with its brilliant ability to model phenomenon has had a far reaching impact in all aspects of our lives and on many fields of study, ranging from biology to economics. This modeling ability has also made Mathematics an extremely useful tool for a wide variety of people ranging from businessmen to engineers. The vast majority of us use computers these days with no clue as to how they work, and similarly Mathematics is used as a tool by practitioners who have no clue as to why the tool works. A view of Mathematics that demands that its utility be demonstrated at all times will also have an adverse effect on Mathematics curricula and the teaching of Mathematics. The trouble is, very little of school Mathematics can be shown to the student to be applicable in a real sense. Most often the examples are rather contrived and meaningless. Furthermore, an attitude that says, "I will learn something only because it is useful" comes in the way of true learning.

A utilitarian approach to Mathematics with benefits to be reaped in the future, while doing mechanical and meaningless calculations in the present, is not going to inspire students. It makes it boring, and Mathematics loses its playful and joyful aspect. As Julian Williams put it "The average student needs emotional and intellectual satisfaction now, not just in five or ten years' time, when they become adults!"

Mathematics as truth and beauty: We now enter esoteric descriptions of Mathematics! All pure mathematicians worth their salt will declare that the reason that they do Mathematics is because it is beautiful. If they are Platonists then they will further declare that they are in search of 'mathematical truth', something to be discovered rather than invented. Who better to express this than G. H. Hardy, the advocate for all pure mathematicians? “A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.” He goes on to say, “The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly Mathematics.”

In my opinion however, the strongest motivation for pursuing Mathematics is experienced at an emotional level. All mathematicians, no matter what their view on the nature of Mathematics, will agree that in the process of creating Mathematics they experience a sense of 'illumination spreading throughout the brain'. Alain Connes, a Fields medalist (the highest honour one can achieve in Mathematics), describes this sensation as follows: "But the moment illumination occurs, it engages the emotion in such a way that it's impossible to remain passive or indifferent. On those rare occasions when I've actually experienced it, I couldn't keep tears from coming to my eyes."

The view of Mathematics that it is akin to a creative art form, and that only those who have tasted the heady joy of discovering Mathematics truly understand it, has the strongest appeal to most people (including myself) who study Mathematics for its own sake, and not only for its applications or other aspects as discussed above. From

time to time mathematicians have lamented the fact that it is because both teachers and students do not truly understand this nature of Mathematics that we have distorted its curriculum and teaching.

However, a view that says that all mathematical experiences that should be similar to experiences of art or music also has its limitations. Beauty in art and music is relatively easily accessible to most human beings, but to see the beauty of Mathematics requires a special connection to it and a fair degree of training. A large part of school Mathematics often does not have a rich enough structure to reveal its beauty; in fact in these years it is problem solving that draws most children to Mathematics. If Mathematics is an art form, then why force all children to learn Mathematics? If we make Mathematics optional based on early reactions to it, are we being responsible to children? Since aesthetics is a matter of taste, must we then allow teachers to fashion their curricula according to their taste? This will surely not satisfy the taste of all their students, let alone help them use Mathematics as a foundation to learn other subjects or earn a livelihood.

Then there is the question of why society should support mathematical activity. Most artists need patrons or find buyers for their art work. Mathematicians do not sell their theorems for a living! Frankly it is because policy makers see Mathematics as a useful tool that most people in the field of Mathematics are able to feed themselves! They either teach Mathematics or 'do' Mathematics which is considered useful for a living. A tiny minority is supported for doing Mathematics for its own sake.

Mathematics for all?

If we insist that Mathematics be part of the core curriculum for all students then we must also make it a fundamental right that all students enjoy learning Mathematics! A renowned Carnatic musician once told me that Carnatic music enjoyed a huge revival some years ago, thanks to the efforts of several young musicians who created and nurtured a broad 'rasika' base. These young people revived the 'sabha culture' in Chennai and hundreds of other small towns and villages throughout South India. Musicians young and old, amateur and accomplished, now have an appreciative audience and can earn a decent livelihood.

Can we create a culture of enjoying Mathematics? Surely this is the only holistic solution to the problems discussed so far. This can happen only if all stake holders really get a feel for the joy and thrill of doing, using and learning Mathematics. This seems a utopian dream given the current state of affairs - unimaginative curricula, poor infrastructure, poorly prepared teachers ("Mathematicians are not interested in teaching children, and teachers are not interested in doing Mathematics," says Paul Lockhart) and a culture of fear and anxiety as far as Mathematics is concerned. But like all revolutions change must begin at both the individual/grassroots and the systemic level.

If we insist that Mathematics be part of the core curriculum for all students then we must also make it a fundamental right that all students enjoy learning Mathematics.

At a systemic level we need to delink mathematical ability from intelligence. We need to help each child discover what they really love, but also learn to love the things they do. We should stop using arcane mathematical problem solving skills as the main criterion for access to resources such as education and jobs. We have to urgently develop a broader base for assessing the aptitude and skills of our young. I am not suggesting a watering down of standards, instead I am asking for a broad based system of evaluation which takes into account the multiple facets of human intelligence, the capacity to be accountable and the more elusive quality of being a sensitive and responsible human being. A radical shift in this area will wipe out cultural anxiety towards Mathematics.

At the level of curriculum, we need to be clear about our goals for Mathematics education. At the minimum we would like everyone to be competent at numeracy, have sufficient critical understanding of data gathering and presentation so that they do not buy into false propaganda, and have the reasoning ability to detect fallacious arguments. For a smaller number of people, the goal would

be one of competence in the use of Mathematics as a tool; for an even smaller fraction, to create new Mathematics (rarely are creative mathematicians the programmed outcome of a system-they turn to Mathematics in spite of any system, as they cannot but do Mathematics!).

The Mathematics curriculum framework outlined in the NCF 2005 is an excellent document and goes a long way in giving very clear guidelines. However, there is an urgent need for a think tank of mathematicians, teachers and educational psychologists to create material keeping these goals in mind. We need to teach numeracy in creative ways so that these skills are mastered and retained. Since they will be used in daily practical situations, these skills should be evaluated through projects and games that simulate relevant situations, rather than through stressful exams. Since Mathematics often builds on itself, concepts need to be revisited but in creative rather than repetitive ways. The whole curriculum should be infused with the philosophy that Mathematics is the 'science of pattern recognition.' We need also to pay special attention to how pattern recognition is assessed. I have had several students who would not appear good at conventional textbook Mathematics, but nevertheless have a very strong spatial sense and are adept at recognizing patterns and solving logical puzzles. Children should have sufficient experience with solving meaningful problems and experience the thrill of having an insight. There is no ready made material available in the market that reflects all these demands. As I have said earlier, it is urgent that we set aside resources to create or at least put together such material in a coherent manner and train teachers to use them effectively.

At the level of the classroom, it is extremely important that a teacher creates a true learning space. For such a space to be created there has to be a relationship of trust and affection between the student and teacher. The teacher must really enjoy doing Mathematics so that his students feel inspired. More importantly, he must help them understand their own fears and resistance to learning, and enable them to take ownership for their own learning. Twenty years of education at Center For Learning has taught us that all these are not romantic pipe dreams, but very much within the realms of possibility.

Suggested Reading

1. John D. Barrow, Pi in the Sky- Counting, Thinking, and Being, Clarendon Press, Oxford, 1992.
2. Jean-Pierre Changeux and Alain Connes, Conversations on Mind, Matter, and Mathematics, Princeton University Press, 1995.
3. Keith Devlin, Lockhart's lament -The Sequel, MAA, Devlin's Angle, May 2008, http://www.maa.org/devlin/devlin_05_08.html
4. Philip J. Davis and Reuben Hersh, The Mathematical Experience, Penguin Books, 1980.
5. Timothy Gowers, Mathematis: A Very Short Introduction, Oxford University Press, 2002.
6. G.H. Hardy, A Mathematician's Apology, Canto Books, Cambridge University Press, 1992.
7. Shashidhar Jagadeeshan, The Nature of Mathematics-an Unfolding Story, Journal of the Krishnamurti Schools, July 2005,Volume 9.
8. Shashidhar Jagadeeshan, On Creating the Right Atmosphere to Teach and Learn Mathematics, Presentation at NCME 2005, New Delhi.
9. Paul Lockhart, A Mathematician's Lament, MAA, Devlin's Angle, March 2008, http://www.maa.org/devlin/devlin_03_08.html10. Davi Mumford, Calculus Reform -For the Millions, Notices of the AMS 44 (1997), 559-563.
10. Mathematics Curriculum in the National Curriculum Framework 2005, NCERT New Delhi.
11. M.S. Raghunathan, The Queen of Sciences: Her Realm, Her Influence and Her Health, 18th Kumari L.A. Meera Memorial Lecture, 2009.
12. William P Thurston, Mathematical Education, Notices of the AMS 37 (1990), 844-850.
13. E.P. Wigner, The unreasonable effectiveness of Mathematics in the natural sciences, Commun.
14. Pure Appl.Math., 1960, 13.

J. Shashidhar received his PhD in Mathematics from Syracuse University in 1994. He has taught Mathematics for over 23 years, and has written a resource book for teachers: “MathAlive!” He enjoys working and communicating with young adults, and developing the senior school programme at Centre For Learning, Bangalore. He can be contacted at jshashidhar@gmail.com