8 Important Concepts in Primary School Mathematics

Kamala Mukunda

From all the varied research into early mathematical ability and learning that psychologists have done, I have chosen for this article two general topics: the fundamental importance of the mental number line, and the connection (often the lack of connection!) between conceptual and procedural learning in primary school. Both are, I believe, topics of particular interest to people working with children aged 6 to 11 years or so, laying the foundation for arithmetic and Mathematics in years to come.

The Mental Number Line

Cognitive scientists have established over the past few decades, pretty conclusively, that as human beings we are 'born to numerate'. Some simple but brilliant work with preschoolers shows that they develop and practice basic, key numerical skills before the age of 4 or 5, spontaneously. The development of these skills is reminiscent of the way children learn language: there seems to be an innate module in our brains that clicks into action given a 'minimum' environmental input. The essential accomplishment of the early years is the proper understanding and use of a mental number line (MNL), used in the act of counting. This is no trivial thing! When a toddler or preschooler counts a set of objects, she invokes no less than five crucial principles.

1. There has to be a one-to-one correspondence between each object and a number name. For eg., you cannot assign more than one object the number 'four'.
2. Yet the number names do not belong to the objects in any way; they can be reassigned. On a recount, you can change all the assignments!
3. Number names are always to be spoken in the same, invariant order. In fact, many toddlers have the wrong number order—one, two, three, five, seven, eight, nine, ten!—but they use it invariantly (till they eventually correct themselves, of course).
4. The final number spoken is always the size of the set.
5. Counting is something you can do with any set of objects, from pins to people.

In time, they use this MNL to compare two numbers to say which is larger, and soon begin to do simple addition using a method called 'counting on'. That is, to add 4 and 2, they start with the larger number 4 on the MNL, and move two units to the right to reach 6. Five year olds have been observed to invent this sophisticated method spontaneously, as they learn to combine their skills of comparison with counting on the MNL.

When children begin formal schooling, they typically should have this informal number knowledge available to understand anything new that is taught. But they do not all begin school equal; studies show (as does any teacher's experience) that in first standard, children vary in their levels of number knowledge. Some students have mastered several number facts, which means that they can quickly recall from memory facts such as '4+2=6' without having to actually re-perform the sum. They also are more likely to use strategies such as counting on, more efficiently, when faced with new problems. Other students are at a disadvantage, not having proper representations of counting on the MNL, therefore not having invented certain strategies, and therefore not having enough number facts at their disposal.

> *Why do we "start from the right, work leftward, shift the products one place to the left, add them all up"?*

Several reasons have been proposed to explain these differences, but psychologists are also working on how to close the gap sooner rather than later, to help weaker students build the foundation they need. The most obvious suggestion is to include explicit instruction of the MNL and its properties in first standard curricula, since it is not generally taught that way. Interestingly, one of the strongest correlates of these initial differences in numerical ability is the socioeconomic status (SES) of the child entering school. Children from lower SES are at a significant disadvantage compared to middle or high SES children at number knowledge when they begin school. Unaddressed, this gap only widens with time. Developmental psychologists Robert Seigler and Geeta Ramani offer one interesting reason for the difference: lower SES children do not have access to the kinds of board games other children routinely play with. The main element of many simple board games (such as Snakes-and-Ladders or Ludo) is a series of numbered spaces, linearly arranged. You move your token along a certain number of spaces, counting as you go, one number for each move. Playing this game, they say, gives preschoolers the right stimuli to develop correct understandings of the MNL.

In a recent study, Siegler and Ramani worked with a large number of children from lower- and middle-SES backgrounds in the U.S. They first replicated the general finding that the lower-SES children perform significantly worse on the following simple numerical magnitude estimation task: given a line with 0 at one end and 100 at the other end, place a third number (say, 37) correctly on the line. Second, they provided the lower-SES children with around 30 brief sessions of playing a very basic board game with just ten linearly arranged spaces. The total time of intervention was only around two hours, and yet post-tests revealed a virtual 'catch-up' of these children! This study needs replication in India, of course. But given the importance of the MNL for early arithmetic, and given the simplicity of the intervention, it is definitely worth investigation.

Marrying Concepts to Procedures

The schism between number recipes and conceptual understanding is widespread. An example of a number recipe would be the long multiplication method, where we are taught to start from the right, work leftward, shift the products one place to the left, add them all up...Primary school Mathematics is filled with such algorithms. But students cannot tell you (because they do not know) why their algorithms work. Interestingly it seems that it is not enough to teach a concept (using concrete materials and so on), then introduce a procedure, followed by tons of drill, rarely revisiting the concept after the algorithm has been introduced. Teachers seem to invest relatively little time and energy into explaining why the procedure works, by connecting it explicitly to the concrete concept. Why do we "start from the right, work leftward, shift the products one place to the left, add them all up"? Explaining the connection once, or even twice, does not seem to be enough given its considerable complexity.

Now, you might wonder why the connection is so important. It turns out that there are strong arguments in favour of explicitly marrying mathematical concepts with procedures. One is that conceptual understanding is itself strengthened in the process. There should in fact be a continuous back-and-forth process between procedural and conceptual learning. Reflecting on the use of a procedure, asking whether and why certain procedures work and others are wrong reinforces our understanding of the concept. Another good reason is that concept and procedure must be interconnected in order for the procedure to be flexibly applied to new problems. And a third reason is that mistakes in arithmetical algorithms can best be corrected through such an understanding, instead of simply through reminders of "how it should be done."

Psychologist Lauren Resnick did a beautiful in-depth study in 1982 of four students in 2nd and 3rd standard, demonstrating conclusively that there was no correlation between their number knowledge and their computational skills. The two sets of knowledge were neatly isolated from each other in the children's minds. Importantly, she found that the students did not spontaneously figure out the connections between the two. She then developed a 'mapping instruction' method for teaching multidigit subtraction, using blocks and number symbols side by side. The blocks were of size 100, 10 and 1, and at each step a particular arrangement of blocks was presented along with the corresponding step of the algorithm. Written computation was a record of the block arrangement, and block arrangements justified the written computation. The entire duration of her instruction method was just 40 minutes! And with this brief and simple intervention, the students were vastly improved both in correct use of the algorithms as well as in explaining in words why things worked a certain way. Again, we see that a relatively small but well designed addition to our teaching can have a powerful impact, especially in primary school settings.

References:

1. Lauren B. Resnick, 1982. Syntax and Semantics in Learning to Subtract. In T. P. Carpenter, J. M. Moser and T. A. Romberg, eds.,
2. Addition and Subtraction: A Cognitive Perspective. Erlbaum.
3. Geetha B. Ramani and Robert S. Siegler, 2008. Board Games and Numerical Development, Child Development, Vol. 79 (2).
4. Siegler has a website that provides almost all his publications for free download: http://www.psy.cmu.edu/~siegler/publications-all.html

Kamala Mukunda studied educational Psychology and then joined Centre For Learning in 1995, where she works with a lively group of colleagues. She teaches Mathematics, Statistics, Psychology, and derives great energy from teaching middle school children. She is the author of “What Did You Ask at School Today?”, a book on child development and learning. She can be contacted at kamala.mukunda@gmail.com

Mathematical Limericks

Limericks are poems which typically consist of five lines and are often humorous or bawdy. Lines 1, 2, and 5 have seven to ten syllables and rhyme with one another while lines 3 and 4 have five to seven syllables and also rhyme with each other.

There was a bright young lad from Madras
Who could not count though in fifth class
His Math teacher tried her best
With tutorials to cram for test
But the bright young lad just could not pass

Entering a discount store is always a tension
Whether to use ratio, proportion or division
Is a price-off percentage of 33
Better than 'buy 2 take 1 free'
Or is it the same? Well, that is the confusion

There was a mathematician who loved his wine
And found the concept of 'Pi' simply divine
He worked hard on 22/7
Mornings and nights, even
To find in the billionth decimal place was a 9

D D Karopady