



**M**athematics is one of the oldest fields of knowledge and study and has long been considered one of the central components of human thought. Some call it a science, others an art and some have even likened it to a language. It appears to have pieces of all three and yet is a category by itself.

According to the National Curriculum Framework (NCF) 2005, the main goal of Mathematics education in schools is the 'mathematisation' of a child's thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise. While there are many ways of thinking, the kind of thinking one learns in Mathematics is an ability to handle abstractions and an approach to problem solving.

The NCF **envisions** school Mathematics as taking place in a situation where:

1. Children learn to enjoy Mathematics rather than fear it
2. Children learn "important" Mathematics which is more than formulas and mechanical procedures
3. Children see Mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on
4. Children pose and solve meaningful problems
5. Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements
6. Children understand the basic structure of Mathematics: arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all of which offer a methodology for abstraction, structuration and generalisation
7. Teachers are expected to engage every child in class with the conviction that everyone can learn Mathematics

On the other hand, the NCF also lists the **challenges** facing Mathematics education in our schools as:

1. A sense of fear and failure regarding Mathematics among a majority of children
2. A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time.

3. Crude methods of assessment that encourage the perception of Mathematics as mechanical computation - problems, exercises, methods of evaluation are mechanical and repetitive with too much emphasis on computation
4. Lack of teacher preparation and support in the teaching of Mathematics
5. Structures of social discrimination that get reflected in Mathematics education often leading to stereotypes like 'boys are better at Mathematics than girls. However the difficulty is that computations become significantly harder, and it becomes that much more difficult to progress in arithmetic.



*The importance of systematic reasoning in Mathematics cannot be over-emphasised, and is intimately tied to notions of aesthetics and elegance so dear to mathematicians.*



The NCF, therefore, **recommends:**

1. Shifting the focus of Mathematics education from achieving 'narrow' goals of mathematical content to 'higher' goals of creating mathematical learning environments, where processes like formal problem solving, use of heuristics, estimation and approximation, optimisation, use of patterns, visualisation, representation, reasoning and proof, making connections and mathematical communication take precedence
2. Engaging every student with a sense of success, while at the same time offering conceptual challenges to the emerging Mathematician
3. Changing modes of assessment to examine students' mathematisation abilities rather than procedural knowledge
4. Enriching teachers with a variety of mathematical resources.

A major focus of the NCF is on removing fear of Mathematics from children's minds. It speaks of liberating school Mathematics from the tyranny of the one right answer found by applying the one algorithm taught. The emphasis is on learning environments that invite participation, engage children, and offer a sense of success.

### Methods of Learning

The NCF says that many general tactics of problem solving can be taught progressively during the different stages of school: abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guess-and-verify exercises, is useful in many problem-solving contexts. Moreover, when children learn a variety of approaches (over time), their toolkit becomes richer, and they also learn which approach is the best. Children also need exposure to the use of heuristics, or rules of thumb, rather than only believing that Mathematics is an 'exact science'. The estimation of quantities and approximating solutions is also an essential skill. Visualization and representation are skills that Mathematics can help to develop. Modelling situations using quantities, shapes and forms are the best use of Mathematics. mathematical concepts can be represented in multiple ways, and these representations can serve a variety of purposes in different contexts.

For example, a function may be represented in algebraic form or in the form of a graph. The representation ' $p/q$ ' can be used to denote a fraction as a part of the whole, but can also denote the quotient of two numbers, ' $p$ ' and ' $q$ .' Learning this about fractions is as important, if not more, than learning the arithmetic of fractions. There is also a need to make connections between Mathematics and other subjects of study. When children learn to draw graphs, they should also be encouraged to think of functional relationships in the sciences, including geology. Children need to appreciate the fact that Mathematics is an effective instrument in the study of science.

The importance of systematic reasoning in Mathematics cannot be over-emphasised, and is intimately tied to notions of aesthetics and elegance so dear to Mathematicians. Proof is important, but in addition to deductive proof, children should also learn when pictures and constructions provide proof. Proof is a process that

convinces a skeptical adversary; school Mathematics should encourage proof as a systematic way of argumentation. The aim should be to develop arguments, evaluate arguments, make and investigate conjectures, and understand that there are various methods of reasoning.

The NCF also speaks of mathematical communication – that it is precise and employs unambiguous use of language and rigour in formulation, which are important characteristics of mathematical treatment. The use of jargon in Mathematics is deliberate, conscious and stylised. Mathematicians discuss what appropriate notation is since good notation is held in high esteem and believed to aid thought. As children grow older, they should be taught to appreciate the significance of such conventions and their use. This would mean, for instance, that setting up of equations should get as much coverage as solving them.

### Organization of the Curriculum

The NCF recommends the following for different stages of schooling:

1. Pre-Primary: At the pre-primary stage, all learning occurs through play rather than through didactic communication. Rather than the rote learning of number sequence, children need to learn and understand, in the context of small sets, the connection between word games and counting, and between counting and quantity. Making simple comparisons and classifications along one dimension at a time, and identifying shapes and symmetries, are appropriate skills to acquire at this stage. Encouraging children to use language to freely express one's thoughts and emotions, rather than in predetermined ways, is extremely important at this and at later stages.
2. Primary: Having children develop a positive attitude towards, and a liking for Mathematics at the primary stage is as important as developing cognitive skills and concepts. mathematical games, puzzles and stories help in developing a positive attitude and in making connections between Mathematics and everyday thinking. Besides numbers and number operations, due importance must be given to shapes, spatial understanding, patterns, measurement and data handling. The curriculum must explicitly incorporate the progression that learners make from concrete

to abstract while acquiring concepts. Apart from computational skills, stress must be laid on identifying, expressing and explaining patterns, on estimation and approximation in solving problems, on making connections, and on the development of skills of language in communication and reasoning.

3. Upper Primary: Here, students get the first taste of the application of powerful abstract concepts that compress previous learning and experience. This enables them to revisit and consolidate basic concepts and skills learnt at the primary stage, which is essential from the point of view of achieving universal mathematical literacy. Students are introduced to algebraic notation and its use in solving problems and in generalisation, to the systematic study of space and shapes, and for consolidating their knowledge of measurement. Data handling, representation and interpretation form a significant part of the ability to deal with information in general, which is an essential 'life skill.' The learning at this stage also offers an opportunity to enrich students' spatial reasoning and visualisation skills.
4. Secondary: Students now begin to perceive the structure of Mathematics as a discipline. They become familiar with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions, and proofs justifying propositions. These aspects are developed particularly in the area of geometry. Students develop their facility with algebra, which is important not only in the application of

Mathematics, but also within Mathematics in providing justifications and proofs. At this stage, students integrate the many concepts and skills that they have learnt into a problem-solving ability. Mathematical modelling, data analysis and interpretation taught at this stage can consolidate a high level of mathematical literacy. Individual and group exploration of connections and patterns, visualisation and generalisation, making and proving conjectures are important at this stage, can be encouraged through the use of appropriate tools that include concrete models as in Mathematics laboratories and computers.

5. Higher Secondary: The aim of the Mathematics curriculum at this stage is to provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that enable such application. A careful choice between the often conflicting demands of depth versus breadth needs to be made at this stage.

On **Assessment**, the NCF recommends that Board examinations be restructured, so that the minimum eligibility for a State certificate is numeracy, reducing the instance of failure in Mathematics. At the higher end, it is recommended that examinations be more challenging, evaluating conceptual understanding and competence.

The NCF's vision of excellent mathematical education is based on the twin premise that all students can learn Mathematics and that all students need to learn Mathematics. It is, therefore, imperative that Mathematics education of the very highest quality is offered to all children.

#### Problem Posing

1. If you know that  $235 + 367 = 602$ , how much is  $234 + 369$ ? How did you find the answer?
2. Change any one digit in 5384. Did the number increase or decrease? By how much?

Source : NCF 2005

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