13 Meaningful Teaching of Mathematics

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One of the major issues with teaching of Mathematics has been that students remain procedural; for instance, they know the algorithm of addition or multiplication but do not know whether solving the problem requires them to add or multiply. Therefore, the challenge before us is to move a student from being 'procedural' to 'proceptual' – the latter meaning procedural plus conceptual, where conceptual indicates understanding and application. Proceptual students display five attributes – let us look at each one of them.

First, proceptual students know the procedure and also understand the concept. Let us illustrate using the familiar procedure of multiplication:

$$\begin{array}{r} 25 \\ \times\ 25 \\ \hline 125 \\ 50\text{x} \\ \hline 625 \\ \hline \end{array}$$

Across different states in India students are told either to put a cross in the unit place, or simply leave a blank, or put a zero. Most of the students follow the procedure mechanically without knowing the 'why' of the procedure. This is what we mean by understanding of the concept.

Second, proceptual students use the most efficient strategy to solve the problem. In the following sum:

$$\begin{array}{r} 299 \\ +\ 21 \\ \hline 320 \\ \hline \end{array}$$

If the student realizes that the problem actually is 299+1 is 300, and plus 20 will make it 320, then she would know that this is a far more efficient strategy than going through the procedure of addition.

Third, student is able to check if their response is reasonable. In the following division:

212 divided by 2

If the students gets the answer as 16 instead of 106, she should be able to see that 16 is not a reasonable answer, and that the answer should be somewhat more than 100.

Fourth, proceptual students have a range of known facts. In the earlier example, that the student knows that 9+1 is 10 and 300+20 is 320 are known facts. Proceptual students have more such known facts. This is one reason why knowing your multiplication tables (with conceptual understanding) is a good idea.

Lastly, proceptual students use known facts to derive other facts. In the same example, 299+21 is 320 is now a new fact derived from known facts.

Having understood the many facets of proceptual students, the challenge now is - how should we teach so that every student of Mathematics becomes proceptual. Here are five teaching strategies which we have found effective in our experience of working with teachers.

> *"It is always a good idea to use more than one method to draw/represent concepts so that the teacher can be reasonably sure that the learner has understood and is comfortable with the concept."*

First, teach from a base of concrete experience. Mathematics is inherently abstract – for instance, as soon as we write 5 to denote five mangoes the learner is making one degree of abstraction. Therefore, as a first step, learner should become comfortable with concrete before making the leap of abstraction. Every learner will take her own time, and we need to cater for this time.

Second, learner should draw or represent concepts. Let us illustrate this strategy through an example. A learner of Mathematics goes through the idea of fractions as one of the first difficult concepts. This strategy urges the teacher to get the learner to draw or represent the idea of fraction in a variety of ways. During our work with teachers we use three different ways of representing fractions. One, we use folding of paper strip to represent fractional numbers. Two, we get them to shade required number of boxes out of the given boxes of equal size to represent a fraction. Third, we get them to mark a given fraction on the number line. The following figures show the three methods of illustrating the fraction 2/5:

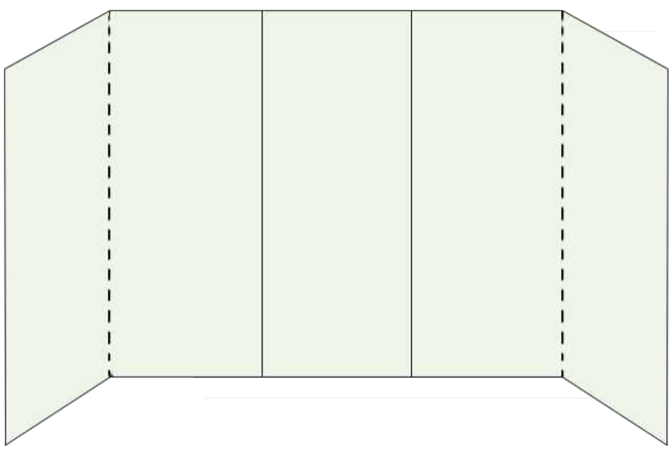

Figure 1

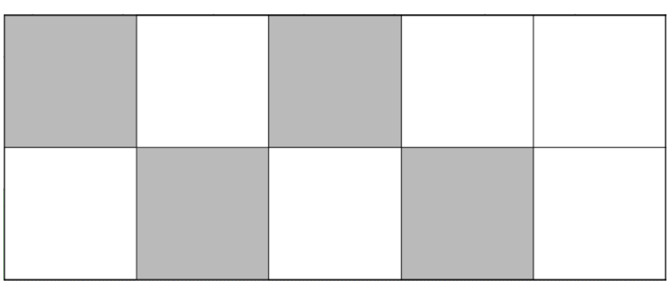

Figure 2

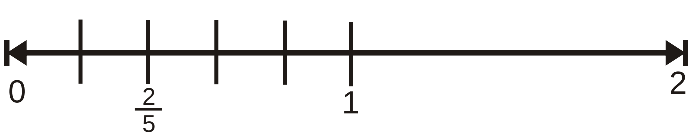

Figure 3

It is always a good idea to use more than one method to draw/represent concepts so that the teacher can be reasonably sure that the learner has understood and is comfortable with the concept.

Third, teacher should verbalize her own strategy while solving a problem. Let us use the earlier problem of division to illustrate this idea. The teacher, after writing 212 divided by 2 in the usual division format, could articulate her strategy by saying – "I first divide the left most 2 of 212 by 2, and the dividend is 1; I subtract 2 from 2 and get zero; I then copy the next digit of 212 which is one; since 1 is less then 2, 2 cannot divide 1 even once and therefore I write zero as the dividend; I then copy the right most 2 and the new number to be divided is now 12; 2 is able to divide 12 completely 6 times, and therefore the answer we get is 106; as you can see if the number was 200 we would have got 100 as the answer, but since the number is 212, which is a little more than 200, we expect the answer to be little more than 100, and therefore the answer seems to be reasonable". By verbalizing her own strategy, teacher models the thinking process for the learner, and also helps the learner start thinking and articulating her own strategy.

Fourth, teacher should use alternative solution strategies to solve a problem and get learners to do the same. Also, get learners to think about which strategy is more efficient. Let us take a simple word problem – "A group of 50 teachers, who are undergoing training, require 10 chart papers for an activity. How many chart papers will be required if 1500 teachers have to undergo training in batches of 50". Now this problem can be solved in a variety of ways. One method could be to work out how many chart papers are required for one teacher (1/5) and then multiply it by 1500 to get 300 as the answer. Second method could be to see that 1500 teachers mean 30 batches of 50 teachers each; further since one batch needs 10 chart papers, 30 batches would require 300 chart papers. Third method could be to use the idea of ratio. If 10 chart papers are required for 50 teachers, how many chart papers would make it the same ratio for 1500 teachers? This would also yield the answer 300. By sharing different methods and discussing their comparative efficiency, we are making the learner become comfortable with concepts.

Fifth, teacher should use the following two questions frequently to engage the learner in

thinking through her response:

1. How did you do that?
2. How do you know you are right?

While the first question forces the learner to articulate her strategy to solve a problem, the second question makes her defend the answer to be a reasonable one. These questions need to be asked of each student even when the answer is right.

We have found in our work with teachers that many of them are not comfortable with the idea of using alternative solution strategies. They feel that it would confuse the learner, and therefore we should stick to one method. We believe that this kind of thinking is what makes the learner a procedural thinker because what she learns is that there is only method to solve a problem and that method becomes for him an algorithm. Therefore, the primary challenge is to make teachers themselves proceptual thinkers in Mathematics.

Meaningful teaching of Mathematics is all about ensuring conceptual understanding for every learner, sharing your own thinking while solving a problem and getting the learner to share hers, and stretching the mind of the learner by solving a problem using alternative strategies.

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Logico- Math Brain Teasers

Three players P, Q and R are participating in a game and made to stand one behind the other in a such a way that P can see both Q and R; and Q can see only R while R can see neither. There are a total of 7 caps, 5 of which are Blue and 2 are Red. One cap each is placed on the heads of these three people from among these 7 caps. First, P is asked if she can tell the colour of the cap on her head. She answers 'No'. The question is repeated with Q who too answers in the negative. When the question is asked of R, she says Yes and proceeds to tell the correct colour. Assuming all three can think logically and they can hear the answers given by the others, what is the colour of the cap on R?

(Hint: Work out all possible combinations)

Use this space for calculation 🙂