3 Pedagogy of Mathematics

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Before we begin certain issues entwined with the word "Pedagogy" need to be pointed out and elaborated. This word is commonly used as a convenient hold all but because of this and in some contexts inspite of this it still can not be completely discussed by itself. To explore its implications some other key elements need to be specified.

Can Pedagogy stand by itself?

The first pre-requisite is the need to know the discipline being considered well. We need to know what it contains and its nature. This means to think about pedagogy of Mathematics, we need to first know what Mathematics is, what it includes, how it functions and then go to other questions. The first level answer to what it contains is: arithmetic and its generalization (i.e. algebra), geometry, statistics, analysis of number system and other such categories. It can be described as abstracting, organizing and generalizing of human experience related to quantity, shape and their transformation. Subsequently it becomes the basic language for building abstract and general ideas in all disciplines. Knowledge and constructs in Mathematics have gone far beyond the initial need of the human society for quantification, measurement and spatial visualization. As an abstract language, it links ideas and concepts in different domains. As it has grown, it has also sought to nurture commonalities across different domains of human experience.

The second pre-requisite is the need to articulate within Mathematics what we are going to transact. The manner in which tables can be memorized is different from the way in which students can be helped to understand how to solve word problems or understand the idea of a variable. Pedagogy is not an epistemic category and cannot help you choose what you want to transact even though it may relate to and even be governed by these choices sometimes and vice verse. This relationship, where it can be seen, is striking and crucial. For example, you cannot help children rote learn tables in a so-called constructivist manner nor have children explore open ended patterns in a classical behaviorist framework.

How do we construct what is to be transacted?

The multifaceted linkages of Mathematics and its abstract nature prompt the NCF to suggest mathematization of child's understanding as a key goal for Mathematics teaching.

This means there needs to be an attempt to help the child abstract logically formulated general arguments, go into organizing her experiences deeply and equip the child to transcend individual events and chance occurrences. In a sense move towards a more general and rational view point. The Mathematics syllabus for the elementary classes

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has to revolve around understanding and using numbers and the system of numbers, understanding comparisons and quantifying them, understanding shapes and spatial relations, handling data etc. In order to understand what aspects of these we need to transact and how we would transact it, the area of Mathematics needs to be understood in a wider perspective. We need to have a broad picture and the entire scope in our minds. This would then need to be narrowed to the classroom and specific choices. For clarity on these we would require a statement in our mind about the reason for these choices.

The capability of solving problems can be considered in many ways. One very obvious way is to get the child to almost copy solutions. The problems given follow the examples. There are no other types of questions leave alone finding ways to address them. A good problem solving task requires being able to locate and find a variety of clues within the problem, find the formulation to solve it and then fulfill the steps. The expectation is development of the ability to solve just one kind of problem in the same way. The way Math is described here is not a system of handling operations but rather the ability to construct and understand algorithms.

This logically leads us to the other question "why do we teach Mathematics?" If children fail to learn to abstract and are not able to follow the logic, do we really need to teach these aspects of the subject to them? Is there a cultural bias to Mathematics or can there even be a genetic bias that implies only some children can learn it? Is abstraction in Mathematics, Science, Philosophy, other subjects including History and Music a non-universal ability? Or is there something peculiar about abstraction in Mathematics? We can all enjoy the rhythm of a beat but to appreciate what is known as classical music or classical dance requires an experience or situations that do not appear to be universally available. Is the ability to generalize and play with numbers and space similar?

In this situation what then should constitute the universal elementary or secondary school curriculum?. What is it that we can expect and want all children to learn such that they do not end up thinking of themselves or being described as incapable? The question asked can be, is it not sufficient for them to know counting numbers and operations on them and a bit of decimal fractions and commonly used fractional numbers? Do we need to insist on making Math abstract and apparently so complex that many cannot follow it? Is the fact that children do not understand a certain kind of Math and are terrorized by it, a result of the way it is taught or is it due to the kind of content covered? Is terror the nature of the subject itself? So there is a complex interplay between the questions – What is Mathematics? And what area of Mathematics is needed and can be transacted in elementary classes? In this we need to also consider whether all of it need to be universally learnt at this stage. We have to spell out (a) why is it needed for that age group, that background and in that historical context for children and (b) can it be learnt by students of that background at that stage given the circumstances of schools and teachers. The choices made need to be able to go through these filters.

It is obvious that it is neither easy to construct these fillers with comprehensive information and arguments and nor is it easy to reach a consensus on implementing and discussing them given the hiatus these abilities seem to provide in the social and economic status accessibility.

As is evident from above content, 'what is pedagogy?' is difficult to address on its own. Its scope and concerns are not articulated very precisely and there is not enough consensus on how it may be defined. There is, however, a common sense understanding that guides the way it is used generally.

What is Pedagogy?

Pedagogy is broadly used to imply the way a subject will be transacted. Described thus there are many obvious components of the word pedagogy. They include classroom transaction and processes, nature and type of teaching-learning materials, assessment system, teacher student relationship, the nature of student engagement, the classroom arrangement etc. This is of course influenced by (and for some people includes) the chosen set of content, information, skills and concepts to be transacted and acquired. Pedagogy needs to worry about the inclusion of all the learners in the learning engagement. This implies the need for an awareness and sensitivity towards diversity and a concern about choices and context in the syllabus. If you carefully consider the manifestations of pedagogy in the classroom, then we know that it is also concerned with the way teachers are prepared, how they are dealt with administratively, the school building, the classroom, the social, economic and political undercurrents existing due to the diversity in the classroom and among teachers. There may also be other systemic and contextual aspects that may influence how transaction takes place. This then becomes a really extended set.

We would here, limit ourselves to some of the aspects. In these a few of the clearly discernible aspects mentioned above will be reiterated as issues that critically influence pedagogic consideration. These include:

(a) Aims of teaching Math
(b) Nature of Mathematics and its key principles
(c) The teacher and her perspective

became definitions and operations. The itemized view of Mathematical ideas implied the narrowing of space for the child to formulate and articulate her own ideas and logic.

Since 'doing' was reduced to a largely mechanical repetition and therefore the 'doing' that stems from exploration, building arguments, developing articulation and definitions to get feedback on them was conspicuously absent. This is not to say that children need to bring out and re-discover the entire human knowledge or they have to discover things by themselves. The knowledge that human society has gathered over time has to be shared, but in a manner that they preserve their freshness of thinking, curiosity and keenness to learn. It cannot mean imposing the hegemony of existing knowledge.

Two views on how to teach Mathematics

In analyzing how Mathematics is taught there are two contrasting views under which programs can be classified. We see classrooms constructed as a combination of these in some proportion. One view is that if you have students practice a lot of sums using algorithms and shortcuts, they eventually start understanding how the algorithm works and may get a sense of why it works. In any case they learn the steps clearly and are able to use it in any context. The nature of questions would, however, be varied.

The other view is that learning Mathematics is about developing an understanding of how the subject is constructed, its basic elements and working out the logical steps that lead to the algorithm and short-cuts in some areas. The child here is expected to be able to develop multiple strategies for problems and also use the algorithms if she finds it appropriate. The argument would not be that this is the best algorithm and has to be learnt by everyone but choose if appropriate. Students can also know, discover and discuss the nature of shortcuts and apply them if they so desire.

There are many examples given for the need for having children learn more than just algorithms. The simplest is addition of two digit numbers and the evidence that very often children introduced to these mechanically, end up viewing them as adding two independent one digit numbers placed in different columns.

There is also a lack of appreciation of the fact that when we multiply any number by a 2 or 3 digit number, the product from the 'tens' digit is not placed directly under the product from the unit place number. It is shifted by putting a cross under the units place. For example:

$$\begin{array}{r} 17 \\ \times\ 23 \\ \hline 51 \\ 34\,\times \\ \hline \end{array}$$

we are not always asked to seek a reason for the shift. There are similar examples from division as well.

Some people argue that the concepts of carry over or borrowing require an understanding of place value and therefore, unless we have children develop reasonable capability in place value they will not be able to do additions and subtractions that require such steps. The essential point here is that the focus is on learning the structure of the subject and the concepts. Once that happens the applications would gradually be learnt by the students. So in these while the eventual goals may be agreed upon, the approach is strikingly different.

Concrete to abstract: What does it mean?

Another aspect of pedagogy is related to the role and nature of materials in the classroom. We generally believe that abstract concepts are acquired through a process of creating, experiencing and analyzing concrete situations. There has been an increasing stress on putting in more and more concrete materials in the Mathematics classrooms. The idea of so-called Math lab has been supported and advocated widely. The feeling is that children understand concepts through the experiences in Mathematics laboratory. This needs to be considered carefully.

It is evident that the idea of using concrete materials and contexts for helping children learn is important. These serve as a temporary model to represent abstract concepts. For example 5 stones are a concrete model for 5 and so 5 chairs. A triangle cut out from card board is a model for triangle as it can portray some key properties of the triangle. It must be recognized that these artifacts do not fully represent the concepts of 5 or the triangle. They are only scaffolds for us to communicate what these terms mean in the initial stages. Gradually learners have to move away from these concrete scaffolds and be able to deal with mathematical entities as abstract ideas that do not lend themselves to concrete representations.

A quadrilateral is closed figure bounded by 4 straight lines. A line is a one dimension infinite string that has no thickness. The point is that an actual line and hence a quadrilateral cannot be represented by even a drawing on the board leave alone by a concrete representation. So while it is important to begin with concrete experiences, gradually the child must articulate using her own language and move on. Mathematics going through the stage of using pictures and then tally marks etc. has to transit to symbols. This is an essential component of learning to do Math. The learning of Mathematics has to culminate in being able to deal with mathematical ideas on their own without any scaffolds. Therefore, when we advocate the Math laboratory for senior schools there is both a pedagogic as well as an epistemic question about whether this is the appropriate direction to proceed in.

The idea of laboratory in Science is to have the students explore some phenomena. She would make observations related to it and then based on the observations attempt to deduce some kind of causal connections. Utilizing many such experiments and data from earlier experiences, the student can attempt generalization and building hypothesis that can be checked by further experimentation. The epistemological touch stone for ideas in Science can be arguably experimental observations and validations. This unfortunately is not true for Mathematics and therefore using the Math lab to have children deduce or prove mathematical statements by measurements or through models, is an epistemic and also a pedagogic error. The attempt at this stage has to be to enable the child to deal with abstract ideas.

Unlike the rich experience of language that the child comes to school with, ideas of Mathematics are not so richly experience based. All children are able to deal with numbers and arithmetic that they need in daily life. They are also able to organize the space around them and carry out spatial transformation to the extent they need. This knowledge is profound and complex. It shows the innate capability of the child to acquire these ideas. All children in any society are able to deal with these ideas. The problem comes when we attempt to transact Mathematics and want them to de-contextualize and abstract the number, shapes transformation, operations and why all these work. The discipline of Math is to be able to talk about abstractions and how relations between abstract quantities can be understood and developed. In the primary classes Social Science and Science are also largely experience based and there is recognition that abstract concepts should not be imposed at this stage. Even in the upper primary classes it is possible to make Science replete with concrete experiences and use the available experiences of the child as well as the experiences provided in the classroom to help her construct a framework of concepts. Mathematics does not allow this easily.

A lot of Mathematics pedagogy depends upon how the teacher engages with children. The classroom atmosphere has to be such that children can participate, articulate their ideas, make mistakes and talk about them without fear. Such an atmosphere will determine the relation children have with Mathematics. There is no one method or one technique that we can recommend for teachers to follow. She has to follow the classroom and create processes that facilitate engagement and dialogue that move forward gradually but can also return to an earlier point and develop again in a different way. The key aspect of Math classroom has to be the recognition that children will develop mathematical ideas and concepts through assimilation with their own previous ideas and experiences and modify them in the process of interactions. Each of us develop our own way of solving problems. It may require exposure to a lot of algorithm and methods but with an openness to create and examine more. They should be able to absorb available ideas and accommodate them in their conceptual framework. The models that anyone of us use or the artifacts a student constructs can help her understand the problem and develop a strategy but would not help everyone. They will be different for each of us.

You cannot help a person learn Mathematics by giving her short-cuts or imposing on her your way of solving problem. Your way may appear very simple, neat and elegant to you but that may not be so for her. We categorize and use ideas in our own ways and use steps that we can think of. It is a doubly difficult task to understand the problem and then also discover the underlying logic of the process you have used to construct the solution.

(d) How children learn Mathematics

(e) The attitude to the subject in society

This will help us derive specific expectations and purposes for different class and age groups. This is what constitutes the syllabus. The first two components have to be informed by the so-called subject, its nature, purpose for human society and for the students for whom the transaction program is being developed. One has to keep in mind the person who is going to transact the learning so as to understand what the aims, expectations and learner backgrounds demand from her. The third is: is there any specific understanding that we need about how this subject is learnt? This will help us construct classrooms that aid learning. The fourth is the prevalent attitude in society about Math- be it teaches, students or parents. All these contribute critically to the pedagogy of the subject.

Teaching Mathematics: Some Approaches

Discussing teaching-learning of any subject requires a basic understanding of how children learn. That should form the basis of our program particularly if each different component of the subject has a character that gives a specific tinge to its learning. The experience of these components for a particular child and the nature of the expectations from her can also be very different in comparison to the other children. For many years, Mathematics learning, like all other learning was considered to be linear and through repeated practice. Whatever was to be learnt had to be broken up into small components and given to children to practice bit by bit. The MLL (Minimum Learning Level) was a crucial example of this approach. In this the pedagogy was claimed to be competence directed.

There is also an expectation from the text book and other materials that for each small element termed as 'competency', there would be one page or one section entirely devoted to it. It was expected that once the child has gone through this she would automatically and surely have developed that part of the competency and needs now to go on to learn the next part. The MLL document itself used the word competency in many different ways. It was used loosely to describe information recall, procedure following, applying formula and in some cases concepts and problem solving as well. As a result of this, it is not clear how the word competency in the MLL document should be unpacked. The on-ground discourse on competency has also not moved forward. In this case Mathematics given its so-called hierarchical nature, its learning seems to be still analyzed in the same framework and conceptualized as bit by bit and through practice of procedures and remembering facts.

Another element that pedagogy is crucially dependent on is the presentation of the teaching learning material (workbook and textbook) and what it expects the child to do and how it suggests the class be organized and assessment made. The material needs to be clear on whom it is addressed to and therefore what it should contain. If the material is for the child then it has to have appropriate spaces, font size, suitable illustrations designed for children and appropriate language.

The textbooks and Mathematics classrooms before the advent of MLL and after the advent of MLL have remained essentially similar due to the fact that students are still being asked to practice algorithms and learn to numerate quickly. Articulation by the child, inclusion of the language of the child and allowing the child to explore and create new approaches to engage with mathematic situations are still not expected and not even accepted in materials. They follow the "consider the given solved example and do some more", approach to Mathematics learning. We may also point out that the mention of a specific competency to be acquired meant the earlier mixed exercises that at least exerted the mind of the child in someway, also got limited to practicising just one option. It was at this time recognition for design, need for illustrations and color in the books emerged so at least the books were different. The principles informing the illustrations, design and other aspects however did not include the need to create space for the child to actively engage her mind.

In the absence of clear articulation, word competency was focused on explanation and telling short-cuts and facts. The key words 'learning by doing' and 'competency', in the context of Mathematics were inadequately explored and insufficiently addressed. Addition was a mere operation and acquiring it was the capability of adding single digit,2 and more digit numbers with no carry over and then with carry over as column additions. In the quest to make Math a doing subject, competency based fractional numbers

Mathematics will be learnt when the student will develop her own strategy, use the concepts and the algorithm in the way she wants. This clearly implies that children must have the opportunity to do lots of problems and solve them in many different ways.

We must expose the learner to these different varieties and develop not only the capacity to construct their own answer but also look and attempt to analyze and comprehend somebody else's answers. They need to be unafraid of making mistakes and confident of articulating their understanding. The implications in the classrooms are that children will work on their own, in groups make presentations on the solutions they have found and construct new problems as well as new generalizations. The classroom has to be such that the child is involved and engaged at each moment.

There has been a lot of talk about constructivism and teaching-learning processes. There have been arguments suggesting that teaching-learning process should be constructivist. This is sometimes interpreted to mean that children should be allowed to follow their own paths and decide what they want to do. It must be emphasized here that like the use of materials in Mathematics the space for the child to articulate her own understanding and building upon it needs to be interpreted in the context of an organized sharing of knowledge with the child and the nature of the discipline. Once the basis of deciding the Mathematics curriculum is arrived at then the classroom and the school has to help the child develop capability in the areas considered important. The teacher cannot ask children what should be done. At best she can construct options that are in conformity with the goals and objectives set out in the program for them to choose from. The notion of constructivism itself and its relationship to Mathematics teaching-learning needs to be explored and analyzed more carefully.

Assessment in Mathematics

An important part of any pedagogical statement is assessment. While there are general principles. The general key principles of assessment such as (a) the purpose of assessment (b) the participation of student in the assessment process (c) the mechanism of assessment (d) the way feedback would be provided to the child.

The manner in which assessment is done at present instills a feeling of fear and purposelessness for most children. Except for those few who are confident of doing well, the others usually want to get over it quickly and scrape through somehow. No one sees a relation between the examination, performance in examination and learning. In Mathematics examinations, particularly, the nature of the tasks given and the manner in which they are assessed lead to children being afraid of not just the examination but even the process of engaging with Mathematics. The entire assessment process is aimed to exhibit what the child does not know rather than to discover what she knows. Concepts of formative, summative evaluation and other such terms do not spell out the purpose, importance and implications of good assessment processes. In recent years we have talked about continuous and comprehensive evaluation, no examination assessment and have argued for the teacher providing extra support to children who lag behind outside the class.

The revocation of the examination, the non-detention policy and the idea of outside the classroom support may appear to be conceptually nice but it is not operationally possible.

Education is a dialogue between school, teachers and the children. If this dialogue is not facilitated with trust, and openness is disallowed it would result in serious distortions in the classroom processes. In Mathematics specifically it is important for the child and the teacher to know what she knows and also have a sense of areas that she is struggling with. The progress of the child needs to be based on what she was able to do earlier. We need to grade the performance of the child in that period rather than grade her against other children. Assessment and expectation is an important part of the requirement to make an effort. The fear of examination cannot take away purpose that assessment serves.

The way society looks at Math is a combination of awe, fear and a passport to success. There are strong beliefs about those who are able to learn Mathematics being more intelligent and have a greater chance and capability to succeed in life.

Mathematics is looked upon as a filter that would separate those who would be moving towards higher intellectual pursuits and those who would take up less intellectual roles in society. The anxiety of occupying the intellectual and technical roles leads parents and teachers to put pressure on students to learn. There is sub-conscious beginning of sorting by declaring many students incapable of learning and therefore helping them by some short-cuts to pass the examinations.

The fear of assessment and subsequent doors that are assumed to open on leaning Mathematics lead to a tense atmosphere in the classroom. The general feeling in the society that it is difficult and has to be such that it can only be done by a few, prevents any attempt to allow children to slowly develop their ability.

It is difficult to conclude this discussion but it is clear that in considering pedagogical aspects of Mathematics it is not merely methods, classroom arrangements and presentations styles that we are talking about. We have to comprehensively look at education and the entire education process, place that in the context of Mathematics, children, parents and teachers along with their aspirations, to move forward on the understanding of pedagogy.

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