Ten Great Mathematicians

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Introduction.

It is not an understatement to say that there are essentially two subjects to study, viz. language and Mathematics. Everything else becomes dependent on them eventually. All of science, including management and social sciences, borrow on Mathematical formulations for concepts. The reason is simple. A precise and concise expression is made possible within the framework of Mathematics. Actually, the abstract layers of certain structural behaviours are increasingly becoming visible in the context of challenging problems from time to time. In this respect, Mathematics is also taken as another language - a symbolic one. Which Mathematicians can be singled out to have induced this transformation?

On a personal note, let me narrate an event. I was presenting a result as a Ph.D. student in Madurai to a visiting renowned Mathematician Nathan Jacobson from Yale University, USA. He listened and finally asked a two-word question "Then what?" I was totally nonplussed. Later, I found an eye-opener in his question. If a result just closes an issue, it is no big deal. If it leaves a door open for further thoughts, it is a great contribution. Which Mathematicians can be recognized using this screener?

There are several worthy names to consider. It is a hard choice to select just ten persons in the history of Mathematics who have influenced its development and changed the perception of the subject over time. My personal choices are Euclid, Fermat, Newton, Euler, Lagrange, Gauss, Cauchy, Riemann, Hilbert and the nom de plume Bourbaki. There are several geniuses who have missed the hit list. One may wonder why I have not included, say, the German physicist Albert Einstein (1879-1955), the Russian Mathematician Andrey Kolmogorov (1903-1987), the Indian Mathematician Srinivasa Ramanujan (1887-1919) or some ancient Indian Mathematicians. Einstein will find his place as one of the ten great physicists as he changed the perception of physics by his relativity theory. Kolmogorov is recognized as the founder of axiomatic probability theory influencing a lot of stochastic methods; but he does not spell a structural influence in Mathematics. Ramanujan is outright marvelous for his strong intuitive contributions; however this genius does not spur a change of perception of Mathematics. As there will be a separate article on ancient Indian Mathematicians, I am not considering them.

Let me take up the ten cases now. I have chosen to list them in the order of the years they lived. In my opinion, the follower in the list was, at least indirectly, influenced by the work of the earlier ones.

> *"It is not the final result of Fermat's Last Theorem by itself that is great; but the techniques that have been developed to get a correct proof for it makes it stupendous."*

1. Greek Mathematician Euclid (around 300 BC)

The first axiomatic treatment in Mathematics was started by Euclid. His structure of proofs in geometry based on postulates about points and lines is a major conceptual contribution. His division algorithm for natural numbers is the most used one and the general Euclidean domains in future Mathematics emerged on that principle.

It is important to assess Euclid's achievements in the historic context of what was perceived as Mathematics before his time. It was a folklore understanding that it is all about practical computations in arithmetic and measurements of geometrical objects. Since around 1700 BC, numerical, algebraical and geometrical methods are attributed to Babylonian Mathematics. Later, a level of practical arithmetic and mensuration developed in Egypt and Italy. Much later, approximately in the 7th to 6th centuries BC, we recognize the Greek Pythagorus and his contributions to geometry. Further on, Hippocrates is credited for circular arcs and areas; and, the logical thinking Zeno (remember his paradox!) introduced the concept of divisibility into infinite parts. During 430-349 BC, Plato's directions into philosophy dominated the conceptual frame of thought. Aristotle (384-322 BC) overlapped Plato's time with his say in formal logic and algebraic notation. By early 4th century BC, ideas of irrational numbers, geometric formulation of areas, algebraic procedures and even integration took roots.

It was Euclid's Elements (see Heath [Hea56]), Appollonius's Conics and Archimedes's Analysis that has been recognized as serious Mathematics for centuries. The codification of geometry can be attributed to Euclid. We can say that Euclid, Archimedes and Appollonius made one great age of Mathematics.

It must be said that a relevant and practical Mathematics was presented in abstraction by Euclid. Such abstracting process has gained currency to transform Mathematics to unbelievable levels over time.

Descriptive geometry as well as algebra dominated the thinking during the first few centuries of AD. Menelaus, Ptolemy and Pappus developed synthetic geometry and Diophantus carried the banner of algebra, influenced by the Oriental thought.

From time immemorial to around 12th century AD, the growth in Mathematical thinking can be felt in terms of (i) computational arithmetic transforming to algebra and (ii) measuring geometrical entities (including studies in astronomy and spherical trigonometry) changing to the synthetic version of geometry like the ingredients of projective geometry. What happened during the next 400 years is mostly unknown. It mostly pertains to the contributions by Indians, Arabs and Greeks passing into the western world of Europe.

As it happened later in history, geometry went beyond Euclid's postulates. Geometry, as learnt today, is enmeshed in algebra.The key was the coordinatisation of geometry. The credit for this goes to Rene Descartes (1596-1650) (see Descartes [Des37]). For the next 200 years, significant achievements in Mathematics were focussed mainly on number theory and analysis. But Riemannian geometry (1894) brought another realm of geometric focus. Some other developments in geometry happened in the 19^{th} century. Along with some significant revelations in foundations through set theory, a thought provoking model of non-Euclidean geometry surfaced. The Russian Mathematicians Nikolay Ivanovich Lobachesky (1792-1856) and the Hungarian Mathematician Janos Bolyai (1802-1860), during the years 1826-1832, questioned Euclid's parallel postulate and obtained the geometric models without this postulate.

2. French Mathematician Pierre de Fermat (1601-1665)

Fermat was the great number theorist. Without going into all his work, especially on prime numbers, let us only touch upon his remarkable scribbling in the margin space of his copy of the book by Diophantus. He mentioned that the space was insuffcient to write his proof about the impossibility of a nonzero integer solution of the diophantine equation $x^n + y^n = z^n$ for $n \geq 3$ unleashed a significant trend thereafter in number theory and algebra to unravel his intended proof. This is familiarly known as Fermat's Last Theorem. The German Mathematician Emst Kummer (1810-1893) gave a false proof of this result based on a mistaken understanding of a certain factorization of $x^n + y^n$ as irreducible and unique. But he corrected his mistake soon and laid the seed for the modern ideal theory in rings.

Many other Mathematicians like Euler, the French Mathematician Adrien-Marie Legendre (1752-1833), the German Mathematician Johann Peter Gustav Lejeune Dirichlet (1805-1859) and Cauchy tried in vain to reconstruct the apparent proof that Fermat might have intended. They succeeded only on the cases n = 3; 4; 5 of Fermat's Last Theorem. With the advent of computers, most sophisticated computer methods were harnessed to verify this result; but they did not succeed. Fermat's work was actually published posthumously in the year 1679. A settlement on this issue took about 400 years. We refer to Edwards [Edw77] and Andrew Wiles [Wil95] to appreciate the vast realm and technicality of Mathematics that it created on this matter.

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3. British Mathematician Sir Isaac Newton (1643-1727)

Newton was the most original contributor among the 17th century Mathematicians. The binomial theorem for rational exponents led to the ideas of infinite series. Obtaining areas by the method of summation of infinite subparts can be attributed to many people, Archimedes onwards to the British Mathematician John Wallis (1616-1703). The idea of differentiation goes back to the French Mathematician Blaise Pascal (1623-1662), who is credited also for the invention of digital calculators. In this background, both Newton and the German Mathematician and philosopher Gottfried Wilhelm Leibniz (1646-1716) attempted the fundamental principle of calculus, viz. the inverse process of differentiation is integration. Leibniz got to calculus independently during 1673-1676 from the works of the British Mathematician Isaac Barrow (1630-1677) and Pascal. Newton got to calculus from the ideas of his teacher Barrow as well as Fermat and Wallis. Both Newton and Leibniz were unable to establish calculus on sound logical basis. Newton included in his work (see Newton [New87]) a short offhand explanation of calculus. This was the greatest of all scientifc treatises; in it can also be found the greatest of scientifc generalizations, viz. the law of gravitation. Newton's aim was to understand nature.

Newton later on explained his calculus on rates of change. In this process, he used the binomial theorem for integral exponents and generalized the same for rational exponents. Leibniz used the idea of infinitesimally small differentials, denoted by $\mathrm{d}x$ and $\mathrm{d}y$ and tried (unsuccessfully) to explain his method in terms of sums and quotients of these. We are using this notation even today. The method of differentials became the mainspring of Mathematical development during the course of 18th and 19th centuries AD. Leibniz's influence on the European continent was far greater than Newton's. Continuity of functions was an intuitively accepted phenomenon in the earlier rounds of thinking. Fortunately for the progress of Mathematics, Newton and Leibniz took for granted that all functions have derivatives. It was difficult enough to obtain an acceptable notion about it. To the differential calculus thus discovered was added the opposite construction of the integral calculus; and the work of Archimedes was belatedly rediscovered. After sometime, it was realized that certain functions might be represented by power series also. Mechanics, even for Newton, led to considering functions as integrals of differential equations. The infinitesimal geometry of curves, extended to surfaces, demanded the introduction of equations with partial derivatives, necessary also for mechanics of vibrating cords. A study of periodic phenomena led to the consideration of trigonometric series. The original classic by the French Mathematician Jean-Baptiste-Joseph Fourier (1768-1830) (see Fourier [Fou22]) contains the basic ideas in this regard, although with no rigour. Thus each forward step in Mathematics engendered, by a chain of creations, the introduction of new entities which were used as tools for other studies and other creations.

Newton's contribution changed the face of both Mathematics and physics.

4. Swiss Mathematician Leonhard Euler (1707-1783)

The Bernoulli family from Switzerland, running through mid 17^{th} century to late 18^{th} century, were fired with enthusiasm for differential calculus. Euler was Johann Bernoulli's student. He, in his treatise (see Euler [Eul48]), brought the concept of function and infinite processes into analysis. Euler was one of the founders of calculus of variations and differential geometry. Both are applications of differential calculus, to cases in which a function depends on another function or curve in calculus of variations and to general properties of curves and surfaces in differential geometry. After Fermat, Euler was the greatest number theorist. The Euler totient function has its claim of importance in number theory. It extended a Fermat result on prime numbers to nonprime numbers also. Euler's formula in radicals for quartic equations is the last such case because Abel proved later that it is impossible to obtain such formulas for fifth and higher degrees. His method of extraction of square roots modulo prime numbers and continuation of certain work by Fermat is another contribution that led Gauss to explore still further. He made strides into continued fractions. In 1779, he also posed a certain conjecture on orthogonal Latin squares; but it took nearly 200 years to prove Euler wrong (see Parker [Par59] and Bose, Shrikhande and Parker [BS59] & [BSP60]). His calculus of quotients of qualitative zeroes and operating with sums of divergent series created furore. Euler also contributed to Mathematical physics.

While Greeks put everything side by side, all aspects of real numbers did not merge with the Greek heritage. While this is so, it seems that the later developments from the beginning of 17th century happened through a perception of compelling need to expand, analyze and rationalize various aspects together. Euler is to be credited for his wide-spread contributions in Mathematics, though he was seen to lack soundness in logical foundations. After Newton and Leibniz, the pioneering 18th century Mathematician was Euler.

5. Italian-French Mathematician Joseph-Louis Lagrange (1736- 1813)

Born in Italy, Lagrange had a career in Prussia and later moved to France. In the context of the French Revolutionary period, we note the establishment of Ecole Polytechnique in the year 1794. Lagrange and Pierre-Simon Laplace (1749-1827) were its first teachers. Lagrange's contribution is in a variety of subjects - algebra, number theory, analysis and mechanics. His interpolation formula is one of the most applicable results in numerical analysis. The result in group theory known today as “the order of an element in a finite group divides the number of elements in the group” is attributable to his work extending Euler's congruence (see on the preceding page) that absorbed Fermat's congruence. He continued Euler's work on continued fractions. He discovered a rule for existence of multiple roots of a polynomial based on the greatest common denominator of the polynomial with its derivative. He is the cofounder, with Euler, of calculus of variations. In his works (see Lagrange [Lag88] & [Lag97]) on mechanics and analysis, he uses deductive logic to bring mechanics in the framework of rigorous Mathematical analysis. Lagrange's theory of functions, the graphical representation of complex numbers obtained in 1813-1814 by the Swiss Mathematician Jean-Robert-Argand (1768-1822) and the imaginary period of elliptic functions explained by Abel in the year 1824 led Cauchy to the so-called integral theorem of complex functions in the year 1825. Lagrange is continuation of Euler with a difference. He brought a level of abstraction that paved a smoother path for future Mathematicians.

6. German Mathematician Carl Friedrich Gauss(1777-1855)

Gauss was one of the first to feel the need for rigour in Mathematics. In the year 1796, at the age of 19, he studied the Euclidean constructions in geometry; arrived at the notions of constructible numbers; and, created a setting for all algebraic numbers of certain types to be considered. The phrases used today like Gaussian integers, Gaussian elimination method, Gaussian distribution, Gaussian channel acknowledge his remarkable contributions in related areas. In 1799, he proved that every polynomial equation with complex coefficients has a root. In the year 1801, he contributed (see Gauss [Gau01]) to number theory in terms of theory of congruences and quadratic reciprocity; these are enormously significant results. His quadratic reciprocity law enabled many numerical calculations without effort. He conjectured that for infinitely many prime numbers p, p-1 is the least number such that p divides 10^{p-1} -1. This conjecture is still unresolved. He provided the basic result, known today as Gauss Lemma, that enables construction of unique factorization domains. He contributed (see Gauss [Gau28]) to differential geometry exploiting the parametric representations.

It is often the case that serious researchers today look backwards to Gauss for inspiration. Another reason for them to do this is to make sure that this giant has not already introduced their ideas.

7. French Mathematician Augustin Louis Cauchy (1789-1857)

It was Cauchy who succeeded in introducing clarity and rigour. He was “forced to accept propositions which may seem a little hard to accept; for example, that a divergent series does not have a sum". He introduced, with precision, the necessary definitions of limit, of convergence, and thus made possible, in a short time, great advances in areas which had been finally clarified.

Actually at the beginning of 19^{th} century, Cauchy closed one period in the history of Mathematics and inaugurated a new one which would appear to be less hazardous. He ruthlessly tested the product of three centuries, establishing an order and a rigour unknown before. He rejected as too vague the habitual appeals to "generality of the analysis" and determined the conditions of validity of statements in analysis with rigorous definitions of continuity, of limits, of different sorts of convergences of sequences or series, which he provided. In the early 19th century, Abel described the state that Mathematics was in when he entered it thus: "Divergent series are completely an invention of the devil, and it is a disgrace that any demonstration should be based on them. One can draw from them whatever one wishes when they are used they are the ones that have produced so many failures and paradoxes. Even the binomial theorem has not yet been rigoursly demonstrated. Taylor's theorem, the base of all higher Mathematics, is just as poorly established. I have found only one vigorous demonstration of it that of Cauchy."

In the year 1825, Cauchy proved the well-known integral theorem of complex functions. At the core foundation level, he offered a construction of the real number system. Also constructed the real number system. Much later in the year 1895, the German Mathematician George Cantor (1845-1918) formulated (see Cantor [Can95]) the theory of sets and introduced how to reason in a framework. In doing so, he justified Cauchy's construction of real numbers to be the same as two other versions created by the German Mathematicians Richard Dedekind (1831-1916) and Karl Weirstrass (1815-1897). We refer to the complete works of Cauchy in [Cau74] for his detailed contributions.

We can say that Cauchy and Cantor started the age of reason. It is Cauchy who laid the strong foundation of Mathematics by insisting on logical reasons to prevail at every level.

8. German Mathematician Georg Bernhard Riemann (1826-1866)

During the mid-19th century, Riemann ruled the development of future Mathematics beyond the imagination of those times. One of his masterpieces of work is complex function theory. His geometric intuition in complex analysis in terms of conformal mappings and the so-called Riemann surfaces, his method of handling differential form for arc length, curvature tensor etc. (in the year 1854) involving ideas found useful and essential later in general relativity and his conjecture (in the year 1859) that is still unresolved going by the name Riemann Hypothesis and which has unleashed an unsurpassed level of Mathematical developments - all these qualify him as the undisputed master and genius.

Riemann's Hypothesis remains one of the most intriguing conjectures in all of Mathematics. It is difficult to describe it without going into some Mathematical vocabulary. It states that all the nontrivial zeroes of a certain complex-valued function of a complex variable, described in terms of an infinite series, must have real part equal to ½. The subject matter of this unresolved conjecture has triggered valuable research in analytical number theory and complex function theory. Riemann Hypothesis is still unproved. But several thousands of zeroes have been verified to fall in place by use of computers.

The Riemann Hypothesis, if found true, would have enormous consequences in number theory. For instance, it would establish a better handle on the nature of distribution of prime numbers. No one would have thought about a connection between prime numbers and analytic functions.

We refer to Riemann's works (see Riemann [Rie90]). We also refer to Laugwitz [Lau99] to find how Mathematics changed since Riemann.

9. German Mathematician David Hilbert (1862-1943)

The German Mathematician David Hilbert (1862-1943) provided the steering wheel for most of the Mathematical developments during the 20^{th} century by his 23 famous problems presented in his address to Second International Congress of Mathematicians at Paris (see Brouder [Bro76]). The surge in ideas contributed by these problems was huge. Though these challenging problems developed future Mathematics to a great extent, some of his expectations were belied later. A notable one was his own student Godel's result (see Godel [Go40]) in logic proving that arithmetic cannot be simultaneously consistent (meaning both a statement and its opposite cannot be true) and complete (meaning either a statement or its opposite is true); that was contrary to Hilbert's intuition. Hilbert's contribution, in the year 1906, to the theory of infinite dimensional spaces is immense. Hilbert is remarkable for his conceptualization of certain earlier trend-setting problems. In the second half of 19th century, problems in electrostatics and potential theory led to a study of integral equations. In the year 1877, the United States Mathematician George William Hill studied matrices of infinite size relating them to perturbation theory of lunar motion. And, in the year 1900, Ivar Fredholm discovered the algebraic analogue of the theory of integral equations. Henri-Leon Lebesgue laid the ground work with measure theory for the later contributions by Banach and Frechet to generalize Hilbert's work. In the year 1922, the Austrian-Hungarian (later territorially Polish) Mathematician Stefan Banach studied the aspects of “geometrizing" the spaces. And the French Mathematician Maurice-Rene Frechet, in the year 1928, generalized these further. Hilbert's contributions in commutative algebra, diophantine equations, number theory etc. are equally noteworthy.

Hilbert's list of problems kindled so much of research activity during the first five decades of 20th century that this period is known as the golden age of Mathematics.

10. Nicolas Bourbaki (1935-continuing)

A group of Mathematicians (mainly French) go together under this pseudonym. Nicolas refers to an ancient Greek hero from whom a French General Charles Soter Bourbaki apparently descended. When Andre Weil was a first year student in Ecole Normale Superieur. He attended a lecture by a senior student, who mockingly presented false theorems and attributed them to various French generals. The last and most ridiculous theorem was named after this Bourbaki. Andre Weil, Henri Cartan, Claude Chevalley, Jean Dieudonne and a few others, all young under 30 years of age, were passionate to bring about changes in future curriculum of Mathematics. Their thought was to publish books from a conceptual and fundamental standpoint. The idea of a forum was born and it was named after Bourbaki, with humour intended.

The first Bourbaki Congress was in the year 1935. The group decided on certain rules for themselves. Membership is limited to 9. Each member would retire at the age of 50. They would meet three or four times a year, each time for a week or two, for a total discussion on the various projects. All members should participate in every project. A member can bring a colleague or even a student as long as these invitees would participate to the same extent that a member is expected to. They should use only axiomatic framework and structural classification of Mathematics to write the books that are agreed to be written. No references could be cited other than a Bourbaki book in line. Everyone would engage in these projects. By turn, each member would prepare a presentation of a topic or a chapter for the book and all others would discuss the material in entirety. Together they would arrive at the final version. There is no authorship and the publishing will go under the name Bourbaki. Bourbaki's aim was to publish high-class text-books quickly. The target size of a book was about 1000 pages running into approximately 10 chapters with the target time set for about 6 months. Their initial list of books is:

(a) Book I. Set Theory

(b) Book II. Algebra

(c) Book III. Topology

(d)Book IV. Functions of One Real Variable Variable

(e) Book V. Topological Vector Spaces

(f) Book VI. Integration

The project moved slowly; only certain chapters were completed by 1942 because of World War II and members going abroad. Roger Godemont, Jean-Pierre Serre, Pierre Samuel, Laurent Schwartz, Samuel Eilenberg were recruited as new members. By 1958, the books were completed. By then, some founding members stepped down and Alexander Grothendieck, Serge Lang, John Tate joined.

Meanwhile Mathematics had grown to a considerable extent, also owing to the inuence of Bourbaki. Members felt that they were not universal Mathematicians to join in all the book projects. However, the decision of Bourbaki was that, even if not universal, their interest to participate in everything was mandatory. No member could stay on the principle of their specialty contribution only. About the rigidity of linear order of arrangement of the topics in books, compromise was made that an organic development would be acceptable without disrupting the unity of Mathematics and structural aspects. The earlier six books needed revisions so new projects could build on them. Bourbaki carried out these and also completed by the year 1980 quite a few chapters in three more books, viz.

(a) Book VII. Commutative Algebra

(b) Book VIII. Lie Groups and Algebras

(c) Book IX. Spectral Theory

Yet another one (Differential and analytic varieties) completed by Bourbaki was just a summary of results and not a full exposition of thoughts. It was just to help the organization of other books.

The various titles of Bourbaki books given here are written in English. Actually, the titles and the work were in French. The first 6 books were later translated into English by Bourbaki.

Bourbaki also publishes several survey articles on advanced topics with their intention to reach out these topics to nonspecialists. These titles emerge after intensive seminars held frequently.

Bourbaki certainly set standards for what a professional Mathematician should know. They brought out books aimed as text-books but they are more like reference books or encyclopedia-cum-treatise-cum-monograph or whatever. What will be the future of Bourbaki? Will their book projects die? Will their seminars and publications take over the main Bourbaki engagement? Will specialists only prevail under Bourbaki banner? The basic starting reason for Bourbaki is the unity of Mathematics. History shows that survival.

Conclusion

Like I said in the beginning, language and Mathematics are the only two subjects that are characteristically fundamental. Through them, we can handle whatever appropriate expressions are needed to convey ideas in essence (abstraction) and lead these ideas to explore further thoughts (concretisation). They have become essential and irreplaceable in our daily life. The selected Mathematicians in this article have amply fortified this and made future embellishments a distinct possibility.

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