2 The Nature of Mathematics and its Relation to School Education

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Background

Mathematics, among all school subjects, enjoys a unique – and paradoxical – status. On the one hand, it is regarded as an essential ingredient of school education. It is taught as a compulsory subject right from Class I to Class X. Moreover, it is often regarded as a kind of touchstone: an educated person is one who knows Mathematics. On the other hand, it is the most dreaded of school subjects, leading to a widespread sense of fear and failure among children. Even adults who have gone through school successfully can be heard to declare: "I could never follow Math in school." (When some of us started the School Mathematics Project at the Centre for Science Education and Communication, Delhi University, in 1992, our aim was to address this fear. For a more recent articulation, see the Position Paper of the National Focus Group on the Teaching of Mathematics, URL http://www.ncert.nic.in/html/pdf/schoolcurriculum/position_papers/Math.pdf)

The above dichotomy raises a number of questions. Some of these are: what is Mathematics and why should we teach it in school? Does the problem with school Mathematics have something to do with the nature of Mathematics, or the way it is taught, or both? Can everyone learn Mathematics up to a point? What Mathematics should we teach in school? How should we teach it?

To attempt to provide answers to all the above questions would be ambitious, even foolhardy. In this article I will focus on some changes that have taken place in the thinking about school Mathematics over the last five decades, and their impact as felt in India in the last few years.

Mathematics for all

Any contemporary discussion on school Mathematics must take into account the context of Universalisation of Elementary Education (UEE). Today, UEE seems to be an attainable target rather than a distant dream. The next milestone of Universal Secondary Education (USE) will surely form a major part of the educational agenda in the coming decade. Thus when we talk of school Mathematics we are talking of something that is addressed to all children.

Can everyone learn Mathematics? The answer, fifty years ago, would perhaps have been a clear NO. Even now, we hear adults talk of children who 'will never be able to learn Mathematics'. How does this face up to the concerns of UEE/USE? Taking a categorical position, the Position Paper mentioned earlier asserts that:

> *Our vision of excellent mathematical education is based on the twin premises that **all students can learn Mathematics and that all students need to learn Mathematics.** It is therefore imperative that we offer Mathematics education of the very highest quality to all children.*

The question which then arises is: what kind of Mathematics teaching can meet the needs of all students? To be able to address this, we need to achieve some clarity about the goals of Mathematics education.

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The aim(s) of School Mathematics Education

Given that all children are going to be learning Mathematics up to Class VIII and perhaps Class X, the main aim of school Mathematics teaching cannot be to produce Mathematicians. It cannot, for that matter, be to help produce scientists and engineers, in spite of the special and important place that Mathematics occupies with respect to these disciplines. What then are the goals of school Mathematics education? The Position Paper says:

Simply stated, there is one main goal— the mathematisation of the child's thought processes.

In other words, the aim is to learn to think about the world in the language of Mathematics, and to develop the kind of thinking that is special to Mathematics. On the other hand, a look at curricula and textbooks in force in the country during much of the last five decades suggests otherwise. It would seem that 'university education', or perhaps 'IIT education' , has dominated the content and style of school Mathematics. No wonder a majority of past and present school goers have no love for the subject!

What is Mathematics, anyway?

If mathematisation of thinking is the main goal of Mathematics education, we need to have some agreement on what constitutes Mathematics. If you ask people at random the question "What is Mathematics?" You will most likely get spontaneous answers "Addition, Subtraction, Multiplication and Division". (On second thoughts or if pressed, people usually add algebra and geometry.) Now these operations on numbers undoubtedly form an important part of Mathematics, but they alone cannot serve to define Mathematics or mathematical thinking. I will not attempt to give a definition; instead, I give you some examples of mathematical thinking.

"The door is between me and the wall."
"There are around fifty toffees in the jar."
"This glass is tall but thin. It will take less water than the wide mug."
"Nineteen and fifteen is ... twenty and one less than fifteen ... that's thirty-four."
"The station is about fifteen minutes if you take the road, but there's a short cut which will get you there in ten minutes."

At first sight, it may seem that the first statement carries no evidence of mathematical thinking. For a pre-school child, however, articulating spatial relationships such as 'above', 'below', 'between', 'beyond' is an important part of mathematisation.

Mathematisation of thought is not an absolute, one-time event. Through school and beyond it, children and even adults continue to Mathematise. On the other hand, our curricula may contain a lot of things that students learn without any accompanying processes, and hence without contributing to the real learning of Mathematics. Here are some examples, which, unless backed up by appropriate classroom processes, could end up being learnt by rote.

"To divide something by m/n, you multiply by n/m."

"The LCM of a and b is a times b divided by the HCF of a and b."

"All triangles with the same base and height have the same area."

The Problem of Abstraction

Young children learn about the world by handling objects. Their introduction to Mathematics therefore, is through the same route. Yet Mathematics, even in Class I, necessarily involves abstraction. Consider a statement from the lowest level of school Mathematics:

"Two and two make four."

This is a statement about two and four, which are abstract entities. The wheels of a bicycle, a pair of socks and two apples have something in common: a property which we can call 'two-ness'. "Two apples and another two apples taken together make four apples" is a statement about the physical world, which can actually be tested – unlike the above abstract statement.

Martin Hughes in his 1986 book "Children and Number" records many conversations with children, which show that children have a "surprisingly substantial knowledge about number" before they start school. However, this knowledge is not couched in the formal language of the Math classroom. A child may correctly count the number of bricks in a box, and predict that if there are eight bricks in it, two more bricks added will make ten bricks in all. Yet the same child has no clue when asked the abstract question: "How many is eight and two?"

Such experiments have subsequently been done by many others, with similar findings. The implication for the classroom is that activities with concrete objects should come before the transition to the formal, abstract language in which mathematical content is usually framed. Moreover, the transition from the informal to the formal should be specifically addressed in our classroom practices.

The Construction of Mathematical Knowledge

Since the basic objects of Mathematics are abstract, we may wonder if they have an existence which is objective and independent of the human mind, or if they are constructs of the mind. This is an issue which philosophers have been debating since at least the time of the philosopher-Mathematician René Descartes (1596-1650). Are numbers, for instance, 'out there', or do they exist only in our minds? The various positions on this are summarised, for example, by Bertrand Russell in his very readable little book "Introduction to Mathematical Philosophy". I will sidestep this discussion for the moment to consider a slightly different aspect of the issue, one which is more directly relevant to the classroom.

It is generally agreed now, following the work of Piaget, Vygotsky and others, that children do not acquire knowledge passively. Rather, each learner actively constructs knowledge for herself. The process of knowledge construction involves interacting with the external world as well as with other people. Thus it does not matter whether mathematical entities have an objective existence or not: we all have to go through the process of constructing them for ourselves.

Although Piaget was not really concerned with school Mathematics, his work bears directly on the learning of Mathematics at the early stages. Constance Kamii has argued, for example, that young children do not discover arithmetic, they re-invent it. At first sight this may seem contrary to the claim that pre-school children have a substantial knowledge of Mathematics, or at least number. However, there is no real contradiction if we remember that children are exposed to many contexts for mathematical knowledge before they enter school.

Is Mathematical Knowledge Unique?

Before we turn to the implications of these considerations for the classroom, we have to address the issue of what Mathematics to teach. Should our curricular choices be dictated by the structure of mathematical knowledge alone? If so, is this structure unique and universal? If this question is posed to a professional Mathematician, the likely answer will be an emphatic YES. However, we must remember that members of the Mathematics research community are a self-defined, closed social group. As argued earlier, the aim of school Mathematics education cannot be to secure for learners membership of this elite group.

Researchers in many countries, including India, have documented many different traditions in Mathematics. Some of these are found in tribal and other isolated communities, while others – labelled 'street Mathematics' – can be seen to co-exist with the formal Mathematics taught in schools. Masons, plumbers and other artisans are often found to use their own, trade-specific, forms of Mathematics.

At a deeper level, the kind of Mathematics that engages the community of mathematicians at any place and time is determined by the other social groups to which the mathematicians belong. Considerations of race, language, nationality and religion cannot be ruled out, even though mathematicians may like to believe they are above and beyond such influences. The picture of Mathematics as a subject that has evolved linearly, largely in the West, from Euclid through Newton to the present day, is one that is increasingly challenged these days.

Implications for the Pedagogy of Mathematics

The above considerations naturally lead to some conclusions on how Mathematics should be taught. Since this volume carries an article on the Pedagogy of Mathematics, I will be brief.

1 Children should be provided contexts in which the learning of Mathematics can take place. These contexts have to be 'realistic' but not necessarily real.
2 In the early classes there should be plenty of opportunity for children to handle concrete objects.
3 Special attention should be paid to the transition to the formal, symbolic mode. Early teaching of algorithms is to be discouraged.
4 Learning basic skills is important, but thinking mathematically even more important.
5 Learners should not be given the impression that mathematical knowledge is a finished product.
6 Overall, the teacher should play the role of a facilitator with each learner engaged actively in the processes of learning Mathematics.

Conclusion

It may appear that issues related to the nature of Mathematics belong to the realm of philosophy, and have little relevance to the teaching of Mathematics in elementary classes. However, as argued above, there is in fact a profound connection. It is important, therefore, for people involved in school Mathematics – teachers, school heads, teacher educators, etc. – to engage at some level with the kind of issues discussed here. How best this can be done remains an open question.

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Logico - Math Brain Teasers

There are 10 smart people who are participating in a game. The 10 players are lined up in a straight line one behind the other so that the last person can see all the 9 others in front of him, the ninth person can see the 8 others in front and so on while the first one can not see any one. The sequence of the 10 players in the line is decided by the Game Master. There are adequate number of Black caps and White caps available. The Game Master will place one cap on the head of each of them. He will then ask each one starting from the last (who can see all others) the colour of the cap on his own head. The player in answer can say either Black or White and nothing else. The person/s who gives the correct answer would be given a prize. The answers can of course be heard by all. The players are allowed some time to discuss and plan their strategy before participating in the game (no tricks like tone change or loudness change etc in answering are permitted). What strategy can they adopt to make sure that the maximum number of them can get a prize? And how many can definitely hope to be get the prize with this strategy?

(Hint: The answer may change if there are 11 prisoners instead of 10)

Use this space for calculation 🙂