15 The Creative Language of Mathematics

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Teaching mathematics at the pre-university level has always been perceived as a task strictly circumscribed by the focus on the crucial public examinations which as students who embark on this course are told by all and sundry 'can make or break your career'. Add to this the rigours of the content heavy ISC (Indian School Certificate) course and the jump from high school mathematics – no wonder innovation and creativity are forced to take second place while preparing students for 'Life!' And yet, can one make sense to adolescents without innovation and creativity? Don't we need to hear what they make of the subject we are teaching? Students like to make their opinions heard and while it may not be possible to devote contact time in class to this, I find that beginning the course with a writing exercise enables me to get to know my students much faster. So the first task I usually set is an essay which encourages students to share their experiences in mathematics and their attitude to it. One such essay was based on the 2001 movie 'A Beautiful Mind'. I used the quote 'There has to be a mathematical explanation for how bad that tie is' and asked students 'Do you agree with John Nash's underlying sentiment that mathematics is all pervasive? Describe some of your encounters with mathematics in unexpected situations'. When a homesick international student spoke of being in a minority and how his emotions were related to numbers, I knew that exercise had done what no counselor could do- get an adolescent boy to speak about his feelings! Many students approach mathematics with negative feelings and it is important to address and accept them instead of treating the subject as a nasty pill which is 'good for them'.

Writing also develops skills such as logical reasoning and the ability to present a valid argument which are highly valued in mathematics. For this reason, I would revisit the writing exercise during the term, often in conjunction with the English teacher. We would assess the exercise for both writing skills as well as mathematical accuracy. One of the exercises that I used was based on the exponential function e^x which has a very special property that the derivative (the rate of change) of the function is equal to the function itself. This function apparently inspired the mathematician Jakob Bernoulli to design his own epitaph 'Though changed, I shall arise the same'. After we studied the exponential function and its inverse, the logarithmic function, students wrote an essay on the phrase 'Though changed, I shall arise the same'. Inventiveness, creativity, individuality and a strong understanding of the nature of change came to the fore- surely we need to develop these skills along with mathematical rigour?

One of the examples used to illustrate beauty in mathematics is a sequence of numbers. While beautiful patterns can be observed, students who are working under pressure often have no time to appreciate what seems to them an endless array which does not connect in any way to their immediate experiences. A particularly difficult exercise is to sum a series and while formulae can be developed and memorised for the sum of an arithmetic or geometric sequence, students of standard 11 are required to develop the sums of sequences in which the differences of the numbers are in arithmetic or geometric sequence! For example, the sequence $1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots$Perhaps you have guessed that the next term is 21 since the difference between the successive terms is 2, 3, 4, 5 and so on. While the enthusiastic mathophile enjoys finding an expression for the n^{th} term of the sequence and the sum to n terms too, it is often hard to teach relentless theory to novices to the area. Which is why, when I taught this topic one December, I found it easy to ask a student to model this problem on the Christmas carol, 'The Twelve Days of Christmas'! Instead of asking the sum to n terms, the challenge to her was to find out how many presents were sent to the singer by her 'true love' – of course without adding up the numbers.

The trigger questions that I used were:

- Is there a pattern you can recognise from the number of presents received each day?
- Can you connect this pattern to one of the sequences you have studied?
- Without counting, can you find a formula which calculates how many presents the girl receives on a particular day? (as the verse describes)
- How many presents did the girl get totally if she got a new set plus a repeat of all the old sets each day? (as the chorus says)

A beautiful bulletin board emerged complete with tiny presents but most valuable of all was the pattern emerging from these presents, one which enabled her to understand the difference between between Tn (the n^{th} term of the sequence) and Sn (the sum to n terms of the sequence) and explain the formula: $Sn - 1/6\ (n)(n+1)(n+2)$. When she calculated the total number of presents to be 364, this sparked a hot discussion on whether one would rather get a present every day of the year (except perhaps one's birthday!) or receive this special treatment on the 12 days of Christmas!

Towards the end of standard 12, students are required to put together almost all the mathematics that they have learnt in functions, differential and integral calculus in the chapter on differential equations. While they regard this as the culmination of their efforts over the last two years, the truth of the matter is that they have a long way to go before they can actually see the math that they study put to actual use. But it certainly helps to have them understand how to use a real life situation and develop a mathematical model from it. And what greater eye-opener than a murder mystery? I have used the following problem several times (once even complete with a gory bulletin board) to introduce differential equations.

At 3.00 a.m. one morning the police were called to a house where the body of a murder victim had been found. The police doctor arrived at 3.45 a.m. and took the temperature of the body, which was 34.5°C. One hour later, he took the temperature again and measured it to be 33.9°C. The temperature of the room was fairly constant at 15.5°C.

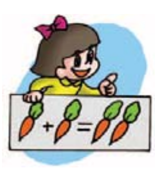
Newton's law of cooling states that the rate of cooling of a body is proportional to the difference between its temperature and the temperature of the surrounding air. Using Newton's law of cooling as a model, estimate the time of death of the victim. (The normal body temperature of a human being is 37.0°C).
Problem Sourced from MEI Structured Mathematics

Clearly, the pre-requisites to solving this problem,

- Using the given data to develop a mathematical model
- Setting down boundary conditions
- Understanding of the symbols used in differential equations
- Knowledge of the method of separation of variables for the solution of a differential equation
- Knowledge of the integral of $\int 1/x\ dx$ and $\int 1/(x - a)\ dx$
- Use of boundary conditions to get constants of proportionality and integration

would, at the start of the unit, be unknown to the students. But the interesting thing was that after each lesson, they would be able to move a little closer to the solution. Towards the end of the unit, estimated times of death would be flying around the class but credit was given only for a full and complete defense of the logic and mathematical steps used to arrive at the correct answer. An important plus is that in the solution to this problem they understand the importance of seemingly minor details such as the constants of integration, the rate of increase of a function and its impact on the time of death, the conveniences afforded by mathematics such as choosing when time $t = 0$, the meaning of dependent and independent variables, etc. No amount of dictated definitions can give students the kind of learning that comes from thinking the problem through.

Of late, technology-savvy teachers have been able to bring a visual element into their mathematics classes using dynamic geometry software and graphing calculators. These not only enable students to do mathematical investigations on their own but have also allowed them to actually understand the meaning of esoteric phrases such as: the limit as n approaches infinity. Of course, designing a mathematical investigation which provides the right amount of scaffolding while allowing the student to work independently requires some skill on the part of the teacher but such activities provide scope for differentiated instruction and exercises multiple intelligences. What better reason for the teacher to exercise her creativity and bring innovation into the classroom?



SNEHA works at the Azim Premji Foundation and also mentors mathematics teachers from rural and city schools. She uses small teaching modules incorporating current technology, relevant resources from the media as well as games, puzzles and stories which will equip and motivate both teachers and students. A teacher of mathematics for twenty years, Sneha resigned from her full time teaching job in order to pursue her career goal of inculcating in students of all ages, a love of learning the logic and relevance of Mathematics. She may be contacted at sneha.titus@azimpremjifoundation.org