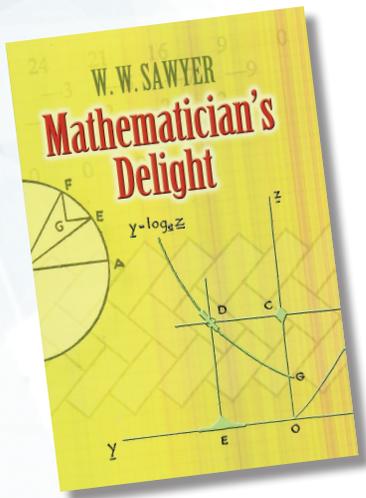


Book Review of W.W. Sawyer's Mathematician's Delight

Reviewed by Mohan. R



It was during lunch on a pleasant day that I was told about this popular book on Mathematics called “Mathematician’s Delight.” I was chatting with a professor who said that his choice to become a mathematician was influenced by this book. The story went like this: When the professor was a teenager, just after high school, during the summer vacation, he found this book and wanted to give it a try. He could follow most of it without much difficulty, and solved most of the exercises which led him to ‘experiment’ with mathematical ideas on his own. When he started his higher secondary school, there was a problem in the mathematics textbook asking him to prove a formula for the sum of the squares of the first n natural numbers using mathematical induction. He neither knew what mathematical induction was, nor how to use it to prove the formula. Yet he knew how to find the formula on his own; not just for the sum of the squares of the first n numbers, but also for sums of cubes, or of fourth powers and so on. He was using the calculus of finite differences that he had learnt from a chapter in the book. This particular incident, he said, made him decide to be a mathematician. I wondered what was in the book that made a teenager not just understand mathematics, but realise that doing mathematics is a rewarding experience.

The book was first published in 1943 by Walter Warwick Sawyer, and is still in print. When I had the book in my hands I had many questions. How is a book written during World War II still relevant today in the age of information? Is the language easy to follow? Who was the intended audience? Were the topics covered relevant (say to a school teacher)?

Most popular mathematics authors try to educate the public on how much fun mathematics can be, on amusing mathematical facts/tricks or on adventures of mathematicians. It is assumed that the reader is already motivated enough to learn these. But

Keywords: mathematization, abstraction, applications, curiosity, commonsense, beauty

that is not often the case. Most of us are afraid of mathematics and we tend to mostly *read about mathematics* than *learn actual mathematics*.

W.W. Sawyer, however, distinguishes himself from the rest. He is known for the mathematics books he authored and his views on how the subject should be taught. His writings seem to arise from a deep understanding of how we learn. He believes,

"Education consists in co-operating with what is already inside a child's mind. The best way to learn geometry is to follow the road which the human race originally followed: Do things, make things, notice things, arrange things, and only then reason about things." (page 17)

In Mathematician's Delight, in addition to 'what and how', he spends quite a lot of time on 'why'. The book is divided into two parts. Part 1 is on 'the approach to mathematics' and Part 2 is on 'certain parts of mathematics'. Part 1 consists of four chapters on how mathematics should be approached and his reasons for doing so. In Part 2, he reconstructs much of school level mathematics along with some advanced topics. Throughout the book, he adopts a questioning approach and engages the reader in mathematical thinking. The topics are built upon the readers' experience of real life situations to which mathematics may be applicable. By starting at the level of simple arithmetic and algebra and then proceeding step by step through graphs, logarithms and trigonometry to calculus and the dizzying world of imaginary numbers, the book makes an effort to constructively encounter the mystery in math.

In Part 1, the author argues that the fear of mathematics is felt not due to the nature of the subject, but due to how it is taught. He gives a brilliant example of how learning is reduced to imitation in a scenario of a hearing and speech impaired child learning to play the piano: the child would have learned the imitation of music without being able to appreciate the music. It makes a point of why there is a need for revolt

against the tradition of dull education. In a chapter on geometry he illustrates how most mathematical ideas occur by putting common sense into action rather than by a stroke of genius.

To illustrate with an example, Sawyer argues how the first mathematicians would have obtained the first theorems and definitions on triangles: The Egyptians knew that a triangle of side lengths 3, 4, and 5 is always a right triangle. But they never bothered to know 'why?' For them it was a god given fact and that fact was useful for building pyramids. But when Greeks learnt the fact from Egyptians, they wondered 'why 3, 4 and 5? why not 7, 8 and 9? What does happen if one tries any three numbers?' This led them to experiment. It would be natural to take three numbers and see what happens to the triangle with those numbers as sides. One would start with fairly small numbers such as (1,1,1), (1,1,2), (1,1,3), (1,2,2), (2,2,2) and so on. As soon as one starts experimenting, they would start to discover some 'facts'. For instance it is impossible to make triangles when one side is bigger than the sum of the other two sides - a simple observation! Hence it can be concluded immediately that (1, 1, n) cannot form a triangle if $n > 1$ - this is just common sense (here n is a natural number). Comparing the triangle of sides (2,2,2) or (3,3,3) with that of (1,1,1) one gets the idea of similar triangles. This way, instead of giving a list of definitions and then another list of examples and theorems, he lets the reader rediscover them more naturally through experiments.

He also discusses why reasoning and imagination play an important role in mathematical thinking. He illustrates the meaning and stages of mathematization (he uses the word abstraction). This vividly brings out how mathematicians first have to work with their 'hands' before working with their 'minds'. To illustrate this he brings up the definition of a straight line. The words straight and line come from Old English for 'stretched' and 'linen' respectively. The first mathematicians were practical men (like carpenter and builders) and they made practical use of straight lines. Yet Euclid's definition of

straight line says they have no thickness. It is mathematization of the already existing idea of how to make straight lines. In laying out a table or in building a house, one is not interested in the size of the rope. Hence, to define straight line, neglect the thickness in order to keep the subject reasonably simple. In the last chapter of Part 1, he acts as a guide and gives a lot of practical advice on how to learn mathematics with available resources. Particularly, he spends a good amount of time explaining how reading around mathematics is better than reading from textbooks. In fact he lists quite a lot of books on different subjects which could be used before learning mathematics directly.

By the end of Part 1, the reader is well prepared to face the subject confidently. Although very engaging, the prose is dense and the author's way of wandering into philosophy might not suit everyone. If the reader wants to dive right into the subject it is recommended that they start the book right away from Part 2, returning to Part 1 whenever it is convenient (except maybe for chapter 2 which deals with geometry).

In Part 2, various topics at school level (and some at an advanced level) are dealt with in detail. Sawyer rightly justifies the claims he makes in Part 1. The reader begins to wonder why her school teacher did not teach like this. What is unusual is the order in which the topics are organized. There is arithmetic, followed by logarithms and algebra, then comes the calculus of finite differences (which is not dealt with at school level), followed by graphs, calculus and then trigonometry, and finally series and complex numbers. There is a clear emphasis on building the new on what is existing already. Thus, for example, the burden of learning all of trigonometry before calculus is removed.

Three distinctive features of the book are (i) developing any mathematical idea in the most 'natural' form, using 'common sense,' (ii) providing the reader with enough hands-on experience that the abstraction follows naturally, and (iii) a great selection of doable exercises

interspersed throughout the book that creates interest to do mathematical experiments.

Here are a few experiments from the book:

1. *If 7 teams enter for a knock-out competition how many matches will have to be played?* The author helps the reader to arrive at a problem solving strategy and then poses the following question: *replace 7 by 2, 176,893 (or by any n).*
2. In chapter 6, the author explains in detail how slide-rules were invented and then asks the reader to make one.
3. In the chapter on Trigonometry, a model is described to demonstrate the meaning of sine and cosine. Then the reader is asked to *make an actual model from the design and then make a table giving the sines and cosines of 5°, 10°, ..., upto 90°, to two decimal places and check the results from printed tables.*

Rather than trying to explain mathematics in a conventional manner, Sawyer engages with those of us who did not get to appreciate the beauty of mathematics in a school classroom by using what was then (and still is, to an extent) a revolutionary approach to explaining maths: tell the student what the problems will be used for, and offer concrete examples, before explaining the mechanics of the concepts. Here is an example from the book: In making a motor headlamp or a searchlight it would be inconvenient to have an electric lamp which spreads out light equally in all directions. One would prefer to have all the light coming out in one direction. It can be achieved by placing a reflector behind the lamp. What shape should the reflector be, if the reflected light is to come out in a perfect beam? Then he explains how to arrive at the answer (parabola) to the above question, in step by step fashion.

However, the book feels a bit outdated as the world has changed enough to feel like a very different place today. The use of imperial units and references to slide rules makes it a bit hard to digest for a modern reader. Hence some of the practical examples and analogies were less

helpful then they presumably once were, and the words used to explain them are stylistically unusual. Most of the examples he uses are in the context of war and there is a subtle glorification of war. To illustrate a few: to introduce negative numbers the author uses examples of a bomb falling into a sea and of an army retreating; to introduce logarithms he supposes a situation in which you are fire-watching on a roof, and have to lower an injured comrade by means of a rope. One starts to wonder if the book was written in and for a battlefield!

Overall the book realizes the objective it originally set to achieve: to dispel the fear of mathematics, and to convince the reader that mathematics is a tool for thinking and a language for common sense. Though this book could be advertised as a popular mathematics book, it is actually a book on mathematics education. It is specifically designed to address the issue of putting mathematics in real-life contexts. More than a student, a teacher would benefit immensely from reading this book. The book is truly a delight to read!



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